Available online at www.sciencedirect.com
ScienceDirect
Insurance: Mathematics and Economics 41 (2007) 163–184
INSURANCE MATHEMATICS & ECONOMICS
www.elsevier.com/locate/ime

Dividend maximization under consideration of the time value of ruin☆

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Received June 2006; received in revised form October 2006; accepted 9 October 2006

Abstract

In the Cramér–Lundberg model and its diffusion approximation, it is a classical problem to find the optimal dividend payment strategy that maximizes the expected value of the discounted dividend payments until ruin. One often raised disadvantage of this approach is the fact that such a strategy does not take the lifetime of the controlled process into account. In this paper we introduce a value function which considers both expected dividends and the time value of ruin. For both the diffusion model and the Cramér–Lundberg model with exponential claim sizes, the problem is solved and in either case the optimal strategy is identified, which for unbounded dividend intensity is a barrier strategy and for bounded dividend intensity is of threshold type.

1. Introduction

The classical optimal dividend problem looks for the strategy that maximizes the expected discounted dividend payments until ruin in an insurance portfolio. For the compound Poisson model, this problem was solved by Gerber (1969), identifying so-called *band strategies* as the optimal ones. For exponentially distributed claim sizes this strategy simplifies to a *barrier strategy*, i.e. whenever the surplus exceeds some barrier level b, all the income is paid out as dividends and no dividends are paid out below that surplus level. In Gerber (1969), the result is first obtained for a discrete version of the model and then obtained for the continuous model by a limiting procedure. Recently, the optimal dividend problem in the compound Poisson model was taken up again by Azcue and Muler (2005), who used stochastic optimal control techniques and viscosity solutions.

The corresponding problem in the case of a diffusion risk process was solved in Asmussen and Taksar (1997) (and in a more general diffusion environment already goes back to Shreve et al. (1984)). Taksar (2000) gives an extensive picture over the above and related maximization problems, where also additional possibilities of control such as reinsurance are treated. Gerber and Shiu (2006) showed that in case the admissible dividend payment intensity is bounded above by some constant $M < c$ (where c is the premium intensity of the surplus process), for exponential

☆ Supported by the Austrian Science Fund Project P-18392.
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0167-6687/$ - see front matter
doi:10.1016/j.insmatheco.2006.10.013

claim sizes a so-called *threshold strategy* maximizes the expected discounted dividend payments (i.e. whenever the surplus is below a certain threshold, no dividends are paid out and above that level the maximal allowed amount is paid). In a diffusion setting, a corresponding result was already established by Asmussen and Taksar (1997).

However, all the strategies outlined above lead to ruin with probability one and in many circumstances this is not desirable. On the other hand, there has also been a lot of research activity on using optimal control to minimize the ruin probability. For instance, for the diffusion approximation, Browne (1995) considered the case where the insurer is allowed to invest in a risky asset which follows a geometric Brownian motion and identified the optimal investment strategy that minimizes the ruin probability of the resulting risk process. For extensions to the Cramér–Lundberg model, see e.g. Hipp and Plum (2000), Gaier and Grandits (2002). The problem of choosing optimal dynamic proportional reinsurance to minimize ruin probabilities was investigated by Schmidli (2001) and optimal excess-of-loss reinsurance strategies were considered in Hipp and Vogt (2003). Combinations of both investment and reinsurance are considered in Schmidli (2002), see Schmidli (in press) for a nice recent survey on this subject.

In this paper we return to the problem of optimal dividend payments, but add a component to the objective function that penalizes early ruin of the controlled risk process. In particular, this additional term can be interpreted as a continuous payment of a (discounted) constant intensity during the lifetime of the controlled process. It will turn out that this choice of objective function leads to a particularly tractable extension of the corresponding available results for pure dividend maximization (in particular Asmussen and Taksar (1997) and Højgaard and Taksar (1999)), and hence considerable parts of the proofs are along the lines of the above papers, however keeping track of the consequences of the additional term in the objective function. The approach should be seen as a first tractable step towards more refined optimization criteria in the corresponding optimal control problems.

The paper is organized as follows. In Section 2, the Cramér–Lundberg model and its diffusion approximation are shortly discussed and the value function underlying our approach is introduced. Section 3 deals with the case of a diffusion risk process and the optimal control problem is solved explicitly, both for bounded and unbounded dividend intensity and the effect of the time value of ruin on the optimal strategy is investigated. It is also shown that if in addition to dividend payouts there is a possibility for dynamic proportional reinsurance, then the optimal strategy from Højgaard and Taksar (1999) is also optimal in our case, just adding a constant term in the value function. Section 4 deals with the above optimal control problem for the classical Cramér–Lundberg process. For exponential claim amounts the explicit solution is obtained, which extends the results of Gerber (1969) and Gerber and Shiu (2006) for unbounded and bounded dividend intensity, respectively. In each section numerical examples are given that illustrate the modification of the optimal strategy with the additional term in the objective function.

After this manuscript was finished, the authors found an unpublished manuscript of Boguslavskaya (2003), who in a financial context used a similar objective function in the diffusion setting for the unrestricted case and solved it using the theory of free boundary problems. However, the approach in Section 3.2 provides a somewhat more intuitive way of proof, using classical stochastic optimal control techniques (and in this case can be interpreted as a detailed exposition of a general approach proposed in Shreve et al. (1984)). In particular this path allows us to extend the results to the Cramér–Lundberg model in Section 4.

Finally, we would like to point out that in a recent paper, Gerber et al. (2006) conjecture that in the case of unbounded dividend intensity, horizontal barrier strategies are optimal for the maximization of the difference of the expected discounted dividends and the deficit at ruin as long as the initial surplus is below the optimal barrier. The results in this paper establish optimality of horizontal barrier strategies for the inclusion of another safety criterion, namely the lifetime of the controlled risk process.

2. Model and value function

Let $(\Omega, \mathcal{F}, P)$ be an underlying complete probability space with a filtration $(\mathcal{F}_t)_{t\geq 0}$ that models the flow of information. Let $W = (W_t)_{t\geq 0}$ be a standard Brownian motion with respect to the given filtration. In this paper two models for the collective risk process are considered. In a first approach the risk process $R = (R_t)_{t\geq 0}$ is described by a diffusion process. Apart from the fact that this assumption simplifies the analysis and leads to structural results, it can also be motivated by an approximation argument towards a compound Poisson model (see Grandell (1977), Iglehart (1969), Schmidli (2004) or Bäuerle (2004)). We denote the drift term by $\mu > 0$ and the standard deviation by σ, then the process with initial capital x is defined via

$$\mathrm{d}R_t = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_t, \quad R_0 = x.$$