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An extended newsboy problem with shortage-level constraints

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Abstract

It is only an order quantity which is decided for a spot selling time in the classical newsboy problem. Both the purchase timing and the time-variant variance of the demand are neglected. When the vendor gives a price discount for early purchase, the buyer purchases the quantities of price discount at the cost of forecast bias. The buyer has to forecast the demand early before purchase ahead of schedule, but it may increase the variance of the forecasted demand, which is a forecast bias. This time-variant variance of the demand is embedded into the model. When the average shortage-level is specified to an upper limit, both the purchase timing and the order quantity are simultaneously considered as the decision variables of the extended newsboy problem in this paper. The resultant outcomes could apply to some cases in the futures contracts. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

We may consider the classical newsboy problem as a scenario on the basis of demand forecasted at time t ahead of time T, and therefore it only decides the order quantity on the tradeoff between over- and under-stocking to minimize cost (or maximize profit).

Since the classical newsboy problem has only one decision variable in it, it is oversimplified. We improved it to be an extended one with two decision variables: (1) when to order (to decide the purchase timing); (2) how much to order (to decide the order quantity). The purchase timing was necessarily considered because it affects not only the purchase cost but also the accuracy of the forecasted demand. The later the purchase timing is, i.e., near the delivering time, the more accurate it is, i.e., the less the variance of the forecasted demand is. The larger variance of the forecasted demand results from forecasting the demand earlier. This characteristic exists mostly in the future contracts. It is different about the assumptions and applications between this paper and the classical newsboy problem, which are summarized as follows:

(1) The classical newsboy problem assumed that the purchase cost per unit is fixed. We assumed that the vendor gives more price discount to stimulate the buyer to purchase earlier for decreasing the inventory level. Based on the different purchase discount at different purchase timing, the model is completely different from

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the other models in the literature [1–4] which is formulated contrarily on the different sales discount at different sales quantity. Although Eeckhout et al. [5] proposed that buyer can reorder the inventory by an expensive purchase price, it was different from this paper which is only one order.

The assumption in price discount is different from the applications in the literature of the classical newsboy problem. For example, Khouja [6] formulated a model on the effects of sale price discount on the demand. Contrarily, we formulated our model on the effects of purchase price discount at different purchase timing on purchase cost.

(2) There is a difference in conducting under-stocking between the classical newsboy problem and this paper. Since the shortage cost includes penalty cost specified in the contract, discount asked by customers for shortage of goods; customers asking some discount for inconvenience; imputation loss; and loss from losing potential customers, it is really difficult to measure shortage cost by experiences. Therefore, some researchers like Aardal et al. [7], Moon and Choi [8] formulated their inventory models with a specified service-level constraint to deal with under-stocking. To widen the application of this model, we formulated the inventory model with a specified average shortage-level constraint.

(3) It was assumed that the variance of the demand was fixed in the literature of the newsboy problem [9–12] namely, the variance is invariant with time. When the buyer forecasts the demand, the nearer the time between delivery and purchase the less the variance of the forecasted demand is; i.e., the less the forecast bias of the buyer is. The effects of the forecast bias on the expected inventory level and the expected shortage of quantity were simultaneously considered in the model of this paper. This tract has not been adopted by other literature relating to the newsboy problem.

The model in this paper could be used in making decisions for some kinds of the futures contracts. It is also suitable for the wholesaler to decide how many and when to buy the seasonal agriculture products. When to order roses and how many roses to order on the eve of Valentine's Day, is a typical example in use.

2. The notation and assumptions

2.1. Notation

2.1.1. Parameters
T delivering timing
δ purchase discount per unit ahead of one unit time
c_t purchase cost per unit at time t; in which c_t is a linear function of $(T - t)$ satisfying $c_T = c$, i.e., $c_t = c - \delta(T - t)$ $0 \leq t \leq T$
v salvage value per unit unsold at time T
h unit holding cost per time

2.1.2. Function
$\phi(z) = 1/\sqrt{2\pi}e^{-z^2/2}$ is a standard normal probability density function
$\Phi(z) = \int_{-\infty}^{z} \phi(x)\,\mathrm{d}x$ is a standard normal distribution function
X_t the demand within period $[0, T]$ which is forecasted at the purchase timing t, satisfying $E(X_t) = \mu$, $\mathrm{Var}(X_t) = \mathrm{Var}$ (demand within period $[0, T]$ forecasted at time t) $= (((T - t)/T)\sigma)^2$.

2.1.3. Decision variable
t purchase timing, $0 \leqslant t \leqslant T$
q order quantity that has been decided by the buyer at time t