PERGAMON

Omega 30 (2002) 45–50

omega
The International Journal of Management Science

www.elsevier.com/locate/dsw

The value of information in the multi-item newsboy problem

Rommert J. Casimir *

Tilburg University, P.O. Box 90153, 5000 LE Tilburg, Netherlands

Received 20 October 2000; accepted 21 September 2001

Abstract

The multi-item newsboy problem is used to determine the value of two types of incomplete information, global information and product-mix information. It is shown that this value depends on the number of products, the existence of a budget restriction and the degree of substitutability.

Keywords: Newsboy problem; Value of information

1. Introduction

In the last decade, the newsboy problem in general and *the multi-item newsboy problem* or *newsstand problem* in particular have been studied in a large number of papers [1]. In contrast to the classical newsboy, who sells a single product, the multi-item newsboy sells several products to a single group of customers. When demand for different products is unrelated, the multi-item newsboy can attain maximal expected profits by maximising the expected profits from individual products [2,3]. To create non-trivial multi-item newsboy problems, three complications have been introduced: performance criteria other than expected profits [2,3], budget or capacity constraints [4–8], and substitutability, which implies that customers may switch to a different product if the demand for their preferred product cannot be satisfied [9,10].

A recent survey shows that, within the context of the multi-item newsboy problem, scarcely any attention has been given to the value of information [1], though this subject has attracted interest in discussions of the single-item newsboy problem [11,12]. In this paper, we try to determine the value of information in the multi-item newsboy problem, and specifically, we are interested in the value of two types of incomplete information, *global information* and *product-mix information* [13]. Global information implies that the combined demand for all products can be accurately predicted, but the distribution over the products is unknown. Product-mix information implies that total demand is unknown, but the distribution over products is known exactly. According to the Bayesian approach, we compute the value of information by comparing expected profits when the information in question is available with expected profits when only the distribution of expected demand is known. Consequently, we do not consider the alternative success criteria discussed in [2,3].

The remainder of this paper is structured as follows. In Section 2, the basic multi-item newsboy problem is outlined and the value of global and product-mix information is computed. In Sections 3 and 4, respectively, the multi-item newsboy problem with a budget constraint and the multi-item newsboy problem with substitutability are discussed. In both instances, we only compute results for the two-item case. Section 5 contains conclusions and suggestions for further research.

In two respects, this research differs from the mainstream of the research on the news-vendor problem [1]. First, it focuses on the value of information in simple models and not on methods to attain an optimal solution in increasingly complex situations, and second, no analytical results are given, but results are computed numerically.

* Fax: +31-13-466-3377.
E-mail address: casimir@kub.nl (R.J. Casimir).

0305-0483/02/$ - see front matter
PII: S0305-0483(01)00054-8

2. The simple multi-item newsboy problem

Throughout this paper, we assume that demand is normally distributed, that demand for different items is independent, that the salvage value for unsold products is zero, and that there is no penalty for demand that is not met. Following [10], we use the notation

- n number of products
- i 1 . . n, an item index
- X total actual demand for all items
- x_i actual demand for item i
- r_i relation between demand for item i and demand for all items $r_i = x_i/X$
- Q_i order quantity for item i
- Q_i^* optimal order quantity for item i
- Q^* optimal order quantity for all n items

For simplicity, we assume that price, cost, average demand and standard deviation of demand for all products are the same, and we use the following notation and data:

- P selling price of a unit ($P = 1$)
- C cost of a unit ($C = 0.7$)
- μ average demand for any item ($\mu = 100$)
- σ standard deviation of demand for any item ($\sigma = 30$)

Though the variability of demand influences the numerical results, it does not change the shape of the resulting graphs, and hence there is no benefit in computing results for different values of μ and σ. Without additional information, the optimal order quantity for each product is equal to the optimal order quantity in the single-item newsboy problem [2,3], and the expected total profit of the n-item newsboy is n times the expected profit of the one-item newsboy.

2.1. Product-mix information

If the product-mix information is known, the n-item newsboy has to determine the overall optimum order quantity Q^*. Subsequently, the optimal order quantity for each individual product is determined from the actual value:

$$Q_i^* = r_i Q^*. \tag{1}$$

If total actual demand is smaller than the optimal total order quantity, then $x_i < Q_i^*$ for all i, and some units of all products are left unsold. Otherwise $x_i \geqslant Q_i^*$ for all i and not all demand can be met. It is not possible that units of one product are left unsold and the demand for another product cannot be met. Hence, total profit of the n-item newsboy is computed by

$$\begin{array}{ll} Q^*(P-C) & \text{if } Q^* \leqslant X, \\ XP - Q^*C & \text{if } Q^* > X. \end{array} \tag{2}$$

As X is distributed with average μn and standard deviation of demand of $\sigma\sqrt{n}$, total profit and optimal order quantity are equal to the profit and optimal order quantity in the one-item newsboy with average demand μn and standard deviation of demand $\sigma\sqrt{n}$. This implies that average profit per product increases with the number of products because of "the well-known phenomenon that product-line forecasts are more accurate than detailed forecasts" [13]. If n nears infinity, the standard deviation of demand relative to average demand nears zero, and hence product-mix information becomes equivalent to perfect information. Numerical results are shown in Table 1 and Figs. 1 and 2.

2.2. Global information

When total demand is known, the n-item newsboy has to determine the optimal order quantity for each individual product with total demand equal to X. To this end, he has to compute the mean and variance of the demand for product A in that instance. If all other products are denoted collectively by B, expected demand for both A and B are normally distributed, and the mean μ_{AX} and variance σ^2_{AX} of A are defined, respectively, by

$$\mu_{AX} = X\frac{\sigma_A^2}{\sigma_A^2+\sigma_B^2} + \frac{\sigma_B^2\mu_A - \sigma_A^2\mu_B}{\sigma_A^2+\sigma_B^2}, \tag{3}$$

$$\sigma^2_{AX} = \frac{\sigma_A^2\sigma_B^2}{\sigma_A^2+\sigma_B^2}, \tag{4}$$

Because B collectively denotes $n-1$ products, $\mu_B = (n-1)\mu_A$ and $\sigma_B^2 = (n-1)\sigma_A^2$. Accordingly, Eqs. (3) and (4) can be simplified, respectively, to

$$\mu_{AX} = X\frac{1}{n}, \tag{5}$$

$$\sigma^2_{AX} = \frac{n-1}{n}\sigma_A^2. \tag{6}$$

Consequently, given total demand of X, the optimum order quantity for each product is equal to the optimal order quantity for the one-item newsboy with average demand of X/n and standard deviation of demand of $\sigma\sqrt{((n-1)/n)}$.

Table 1 and Fig. 1 show the average profit per product of the n-item newsboy for n in the range 2–10 with different types of information. The numbers have been computed by multiplying the probability that a given number of units is sold with the profit that is attained in that case. As stated in the introduction, the profit of the n-item newsboy without additional information does not depend on the number of products.

The value of product-mix information and global information is plotted in Fig. 2. It is computed by subtracting the profit without additional information from the profit with the pertinent information. As expected, Fig. 2 shows that the value of product-mix information increases with the number of items, whereas the value of global information decreases with the number of items.