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A binary solution method for the multi-product newsboy problem with budget constraint

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ABSTRACT

Multi-product newsboy problem (MPNP) with budget constraint is a classical inventory control/management problem. However, solution methods for MPNP under general demand distributions are limited in the current literature. In this paper, by analyzing properties of the optimal solution to the MPNP with a budget constraint, we develop a solution algorithm for the constrained MPNP. The proposed algorithm is binary in nature, and is applicable to general types of demand distribution functions, discrete as well as continuous. For continuous demand distribution function, our approach can obtain the optimal or near optimal solution to the constrained MPNP with polynomial computation complexity of the $o(n)$ order. On the other hand, for discrete demand distribution functions, it can effectively provide good approximate solution. Numerical experiments are presented to show the performance of our method.

1. Introduction

Multi-product newsboy problem (MPNP) with budget constraint, introduced firstly by Hadley and Whitin (1963), is a classical inventory control/management problem. Hadley and Whitin (1963) presents a solution method to the constrained MPNP, which encounters difficulties, particularly when the number of products is rather large. After Hadley and Whitin's early work, many researchers have developed different solution methods for MPNPs with different application background. Khouja (1999) presents a good literature review on these researches.

Erlebacher (2000) develops heuristic solutions for the MPNP with one capacity constraint. He begins by proving the optimality of the order quantities for two special cases, then he proceeds by developing heuristics for a few specific probability distribution functions. Vairaktarakis (2000) develops several minimax regret formulations for the MPNP with a budget constraint. His approach can obtain the optimal solution only when the values of demand are intervallic or discrete. For MPNP under general demand distributions, these approaches provide heuristic solutions rather than the optimal solutions.

Abdel-Malek et al. (2004) develops Lagrangian-based methods, which yield the optimal solution to the problem when the demand is uniformly distributed, and near optimal solution when the demand is other continuous distributions. As it is pointed out by Abdel-Malek and Montanari (2005a), most of the existing Lagrangian-based methods do not pay much attention to the lower bounds of the order quantities (non-negativity constraints), e.g., Abdel-Malek et al. (2004), Ben-Daya and Raouf (1993), Erlebacher (2000), Gallego and Moon (1993), Khouja (1999), Moon and Silver (2000), and Vairaktarakis (2000). This negligence, as observed by Lau and Lau (1995, 1996) could lead to infeasible order quantities (negative) for some of the considered products.

To address non-negativity constraints of the order quantities, Abdel-Malek and Montanari (2005a) extend the research of Abdel-Malek et al. (2004), and propose a modified Lagrangian-based method by analyzing the solution space. Their approach, however, can be applied

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to obtain the optimal solutions only for special demand distributions, e.g., uniform and normal demand.

In summary, the existing methods for the capacitated newsboy problems have the following limitations: (1) many existing Lagrangian-based models could lead to infeasible order quantities (negative) because of relaxing the lower bounds of the demand (Lau and Lau, 1995, 1996); (2) current solution methods can only solve the optimal solution for the special cases (e.g., Erlebacher, 2000; Abdel-Malek et al., 2004); (3) for general demand distributions, the existing methods can only provide approximate or heuristic solutions (e.g., Erlebacher, 2000; Abdel-Malek and Montanari, 2005a).

In this paper, by analyzing properties of the optimal solution to the MPNP with a budget constraint, we develop a solution algorithm for the constrained MPNP. The proposed algorithm can overcome some of the aforementioned limitations of the current methods. Additionally, it is applicable to both types of demand distribution functions, discrete as well as continuous.

The reminder of this paper is organized as follows. We describe the constrained MPNP problem and the optimal solution to the unconstrained MPNP in Section 2. In Section 3, by presenting the properties of the optimal solution to the constrained MPNP, we develop a binary solution method for the constrained MPNP under general demand distribution. Numerical examples are illustrated in Section 4. Section 5 briefly concludes the paper. The proofs are presented in Appendix A.

2. The constrained MPNP

2.1. Mathematical model

In order to introduce a clear description of the constrained MPNP, we first present the definition of the notations in Table 1.

The model of the constrained MPNP can be expressed as

$$\operatorname{Min} E = \sum_{i=1}^{n} E_i(x_i) = \sum_{i=1}^{n} [c_i x_i + h_i E(x_i - D_i)^+ + v_i E(D_i - x_i)^+], \quad (1)$$

Table 1
Notations.

Notations	Definitions
n	total number of products
i	product index
v_i	cost of revenue loss per unit of product i
h_i	cost incurred per product i for leftover at the end of the specified period
c_i	cost per unit of product i
x_i	amount to be ordered of product i which is a decision variable
D_i	random demand of product i
$f_i\ (D_i)$	probability density function of demand for product i
$F_i\ (D_i)$	cumulative distribution function of demand for product i
E_i	expected cost function of product i
E	total expected cost function
B	budget function
B_G	available budget

subject to

$$B = \sum_{i=1}^{n} c_i x_i \leqslant B_G, \quad (2)$$

$$x_i \geqslant 0, \quad i = 1, 2, \ldots, n \quad (3)$$

The total expected cost in Eq. (1) is the sum of purchase cost, expected overage and underage costs of all products. Eq. (2) is a budget constraint, and Eq. (3) expresses the non-negativity constraints on order quantities.

2.2. Optimal solution to the unconstrained problem

By relaxing the budget constraint, the constrained MPNP becomes an unconstrained problem. The optimal solution of the unconstrained problem can be solved by taking the partial derivatives of E with respect to x_i ($i = 1,2,\ldots,n$) and setting them to zero (Hadley and Whitin, 1963).

If the distribution of demand D is continuous, we can interchange the derivative and the expectation operators, it follows that

$$\partial E/\partial x_i = c_i + h_i E\delta(x_i - D_i) - v_i E\delta(D_i - x_i), \quad i = 1, 2, \ldots, n, \quad (4)$$

where $\delta(x) = 1$ if $x_i > 0$ and zero otherwise. Since $E\delta(x_i - D_i) = F_i(x_i)$ and $E\delta(D_i - x_i) = 1 - F_i(x_i)$, Eq. (4) can be rewritten as

$$\partial E/\partial x_i = c_i - v_i + (h_i + v_i)F_i(x_i), \quad i = 1, 2, \ldots, n. \quad (5)$$

Setting the derivative to zero reveals that

$$F_i(x_i^*) = (v_i - c_i)/(h_i + v_i) = \theta_i, \quad i = 1, 2, \ldots, n. \quad (6)$$

Note that the expected cost is a convex function of x_i, ($i = 1,2,\ldots,n$) since $\partial^2 E/\partial x_i^2 = f_i(x_i) \geqslant 0$. If $F_i(x_i)$ is strictly increasing, $F_i(x_i)$ has an inverse function; otherwise we define the inverse function and yield the unique optimal solution as follows:

$$x_i^* = F_i^{-1}(\theta_i) = \inf\{x_i \geqslant 0 | F_i(x_i) \geqslant \theta_i\}, \quad i = 1, 2, \ldots, n. \quad (7)$$

If the distribution of demand D is discrete, by writing $E(x_i - D_i)^+ = \sum_{j=x_i}^{\infty} (x_i - j)\Pr(D_i = j)$ and working with the forward difference, it is easy to see that the optimal solution is given by $x_i^* = \min\{x_i \in \Phi | F_i(x_i) \geqslant \theta_i\}$, where Φ is the support of the discrete demand D (you can refer to mathematics book).

3. The binary solution method

Before presenting the binary solution method, we first present some properties of the optimal solution to the constrained MPNP.

3.1. Properties of the optimal solution to the constrained MPNP

To analyze properties of the optimal solution to the constrained MPNP, in the spirit of Hua et al. (2006), we