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A newsboy problem with a simple reservation arrangement

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ABSTRACT

Newsboy problems have always been an important issue in inventory management. A number of methods have been proposed to solve such problems based on different objectives or considerations. Different from the existing studies, this paper presents a newsboy model with a simple reservation arrangement by introducing the willingness rate, represented as the function of the discount rate, into the models. Mathematical models are developed, and the solution procedure is derived for determining the optimal discount rate and the optimal order quantity. Through the numerical example, we demonstrated the varied profits yielded from the models considering the reservation arrangement, depending on the number of consumers who accept the reservation policy. The profits derived are greater than those from the classical newsboy model, due to the consideration of the reservation.

1. Introduction

Inventory management is an important task in business operations. The classical newsboy problem deals with the purchasing inventory problem for single-period products, such as perishable or seasonal goods. For such problems, a business manager should determine product order quantity at the beginning of each period. Products cannot be sold (or the price is trivial) in the next period if the order quantity exceeds actual demand; however, there is an opportunity cost of lost profit in the reverse situation. Therefore, the determination of the order quantity is critical in the classical newsboy problem.

In the real world cases, a reservation policy is usually adopted by business units for some commodities, including newsboy-type products, since it can reduce the demand uncertainty and therefore profits could increase. The motivation for a consumer to make a reservation may be complicated, including both psychological and economic factors. Regarding the economic factors, a discount could be an important incentive, and in general, the higher the discount the higher the willingness of a customer to make a reservation. In other words, the discount rates have an influence on the sale quantity and therefore the order quantity. However, a higher discount rate could lead to lower profits, or even cause a loss, despite the higher sales. Accordingly, from the business unit's side, the determination of the optimal order quantity as well as the optimal discount rate will influence the expected profit arising from the solution to a problem.

For classical newsboy problems, many researchers have proposed a number of approaches to determine the optimal order quantity in order to deal with the uncertainty of demand (e.g. Anvari, 1987; Fathi & Nuttle, 1987; Hadley & Whitin, 1963; Ismail & Louderback, 1979; Kabak & Schiff, 1978; Khouja & Mehrez, 1996; Lau & Lau, 1988; Lau, 1980; Lau & Lau, 1995; Li, Lau, & Lau, 1991; Sankarasubramanian & Kumaraswamy, 1983; Silver, Pyke, & Peterson, 1998). Almost all of these studies consider the product quantity demand as a random variable to determine the optimal order quantity in order to maximally achieve the anticipated objective, such as expected profit, expected utility, the firm's market value, and the probability of reaching the expected profit level. In addition, some researchers have extended the classical newsboy problem based on various considerations. For example, considering the products with a long-selling period and highly volatile stochastic demand, Chung and Flynn (2001) introduced reactive production into the classical newsboy problem. Khouja and Robbins (2003) addressed the important effects of advertising on sales, assuming that the mean demand is increasing and concave with respect to the advertising expenditure. Weatherford and Pfeifer (1994) introduced the system of advance booking of orders, and indicated that the optimal discount rate is an important factor to maximize the expected profit. Unlike the existing literature, this research proposes a newsboy problem with a simple reservation arrangement by introducing the willingness rate, represented as a function of the discount rate, into the models. The solution procedure is derived for determining the optimal discount rate and then the optimal order quantity for such a problem.

This paper is organized as follows. In Section 2, we briefly introduce the model of the classical newsboy problem. The model with the reservation arrangement is then presented, and the optimal

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solutions are derived in Section 3. In Section 4, a numerical example is given to demonstrate the feasibility of the proposed model. Finally, conclusions are provided in Section 5.

2. The classical newsboy model

The total profit function (Z_c) in the classical newsboy model depends on the demand quantity and order quantity, and is formulated as (Silver et al., 1998)

$$Z_c = \begin{cases} (s-v)x + (v-c)Q_c, & x < Q_c \\ (s+p-c)Q_c - px, & x \geqslant Q_c \end{cases}, \tag{1}$$

where

c	unit cost,
s	unit selling price
v	unit salvage value ($v < s$)
p	unit shortage cost
x	demand quantity during a period, which is a random variable following a probability distribution
m_x	the median of random variable x
Q_c	order quantity (decision variable).

Since the demand quantity is a random variable following a probability distribution, the expected profit is

$$\begin{aligned} E(Z_c) &= \int_0^{Q_c} ((s-v)x + (v-c)Q_c) f_x(x)\mathrm{d}x \\ &\quad + \int_{Q_c}^{\infty} ((s+p-c)Q_c - px) f_x(x)\mathrm{d}x \\ &= (s-v)\mu_x + (v-c)Q_c - (s+p-v)\int_{Q_c}^{\infty}(x-Q_c)f_x(x)\mathrm{d}x, \end{aligned} \tag{2}$$

where $f_x(x)$ is the probability density function (pdf) of the random variable. Taking the first derivative of $E(Z_c)$ with respect to Q_c, the maximum value of $E(Z_c)$ occurs at Q_c^* which satisfies $\mathrm{d}E(Z_c)/\mathrm{d}Q_c = 0$. The optimal order quantity can be obtained as

$$F_x(Q_c^*) = (s+p-c)/(s+p-v), \tag{3}$$

where $F_x(\cdot)$ is the cumulative density function (cdf) of random variable x. For a given parameter set of s, p, c, and v, the optimal order quantity Q_c^* can be easily determined with specified mean μ_x and the standard deviation σ_x, as follows

$$Q_c^* = F_x^{-1}\left(\frac{s+p-c}{s+p-v}\right). \tag{4}$$

From the above equation, if $\frac{s+p-c}{s+p-v} < \frac{1}{2}$, then obviously the inequality $Q_c^* < m_x$ holds; otherwise, it is reversed.

3. The newsboy model with a simple reservation arrangement

In order to develop the newsboy model with a simple reservation arrangement, firstly we make the following assumptions.

(1) Customers only have one channel to acquire the product, i.e., they must purchase either via the reservation or from the business unit directly.
(2) The total order quantity has to be determined prior to the reservations.
(3) Once the order is released, the reservation cannot be canceled.
(4) A discount rate is given to consumers who make a reservation for the newsboy-type product for the purpose of promotion, with the consideration that the total demand is not influenced.
(5) The higher the discount rate, the higher the willingness to make reservations. The willingness rate β is defined as the proportion of possible customers, those who actually buy the product in this paper, and this is a function of the discount rate α, i.e., $\beta = g(\alpha)$, $\alpha,\beta \in [0,1]$. In other words, the total order quantity is enough for the demand from the reservation, since β is less than or equal to one.

3.1. Formulation and solution procedure

The total profit of the newsboy model considering reservation policy, Z_r, is composed of the profit Z_{r1} from the reservation and the profit Z_{r2} from the usual sales. Since the willingness rate for reservation would be a function of the discount rate, the discount rate will have an influence on the order quantity and the total profit. Therefore, the order quantity and discount rate will be the decision variables for maximizing the expected total profit in the model. Let the notations of c, s, v, p, and x be the same as those in the classical model. The profit from reservations is thus

$$Z_{r1} = \beta x(s(1-\alpha) - c) = g(\alpha)x(s(1-\alpha) - c), \tag{5}$$

and the profit from the usual sales is

$$Z_{r2} = \begin{cases} (s-v)y + (v-c)Q_r, & y < Q_r \\ (s+p-c)Q_r - py, & y \geqslant Q_r \end{cases}, \tag{6}$$

where Q_r is order quantity for the usual sales, and y is the random variable of demand quantity for the usual sales with $y = (1-\beta)x$. Note that the random variable of demand quantity for the reservation is $g(\alpha)x$ in (5). Obviously the total expected profit is

$$E(Z_r) = E(Z_{r1}) + E(Z_{r2}).$$

From (5) and (6), the corresponding expected profit can be formulated as

$$E(Z_{r1}) = g(\alpha)\mu_x(s(1-\alpha) - c) \tag{7}$$

and

$$\begin{aligned} E(Z_{r2}) &= (s-v)(1-g(\alpha))\mu_x + (v-c)Q_r - (s+p-v) \\ &\quad \times \int_{Q_r}^{\infty}(y-Q_r)f_y(y)\mathrm{d}y, \end{aligned} \tag{8}$$

respectively. Using $y = (1-g(\alpha))x$, (8) can be expressed as

$$\begin{aligned} E(Z_{r2}) &= (s-v)(1-g(\alpha))\mu_x + (v-c)Q_r - (s+p-v) \\ &\quad \times \int_{Q_r(1-g(\alpha))^{-1}}^{\infty}((1-g(\alpha))x - Q_r)f_x(x)\mathrm{d}x. \end{aligned} \tag{9}$$

From (7) and (9), the willingness function $g(\alpha)$ should be determined beforehand for resolving the expected profits. According to the definitions of α and β ($\alpha,\beta \in [0,1]$), the definition of a willingness function should satisfy the following axiomatic requirements: (1) $g(\alpha = 0) = 0$ and $g(\alpha = 1) = 1$ (boundary condition), (2) If $\alpha_1 \geqslant \alpha_2$, then $g(\alpha_1) \geqslant g(\alpha_2)$ (monotonicity), and (3) g is a continuous function. A willingness function could have various types which satisfy the required axioms in the real world, because the consumers' attitude for certain products could be different. As an example, the functional relationship of β and α could be a simple linear form as $\beta = g(\alpha) = \alpha$. Four different willingness functions are given as examples in Appendix. Using these functions in (7) and (9), it can be proved that there exists an optimal value of the total expected profit $E(Z_r)$, as described in Appendix.

In order to maximize the total expected profit $E(Z_r)$, the optimum Q_r and α can be found by carrying out the usual mathematical optimization processes. Firstly, the order quantity Q_r for the usual sales is calculated as

$$Q_r = (1-g(\alpha))F_x^{-1}\left(\frac{s+p-c}{s+p-v}\right). \tag{10}$$