A multiple-item budget-constraint newsboy problem with a reservation policy

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**Abstract**

In contrast to the classical newsboy-style problem, this paper develops a model for the multiple-item budget-constraint newsboy problem considering a reservation policy to meet marketing needs. A discount rate is provided to those customers who are willing to make a reservation. In addition to the demand from the original customers, extra demand is also included in the model due to the motivation of the discount rate. A solution algorithm, namely the MCR algorithm, is proposed to solve the problem. The proposed algorithm can actually be considered as a generalization of the classical newsboy-style problem. The MCR algorithm not only provides a business unit with the optimal order quantity, but also the discount rate necessary to achieve the maximal total expected profit under a limited budget. From the illustrated example, it is shown that the expected profit from the proposed model is greater than that from the classical model, due to the consideration of reservations, even though the budget constraint is tight. The increase in the expected profit could be treated as an information value from the willingness function. Although the proposed model is developed for multiple-item problems, it also can be applied to single-item ones. From the sensitivity analysis, the application of the single-item problem to the proposed model can still obtain greater expected profit than the classical model under different budgetary levels.

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1. Introduction

Single-period commodities, such as newspapers, milk, magazines, flowers, Christmas trees, pop CDs, and so on, are common in daily life. These kinds of products have high/low value at the beginning/end of the selling period. This selling period is varied, depending on the property of the product, such as a day, week, month, season or even a year. These products usually have expiration dates after which they cannot be sold or can only be sold at a trivial price. Therefore, it is a very important task for business managers to determine the order quantity of such products at the beginning of each period to maximize the total profit.

The classical single-period model, also known as the newsboy problem model, helps to solve the optimal order quantity to minimize the total cost or maximize the total profit [1,2]. This model has an assumption that if any inventory remains at the end of the selling period, the price of the good is trivial. Conversely, if there is any unsatisfied demand at the end of the selling period, it causes an opportunity cost and the loss of some profit. Due to the uncertainty of demand, researchers usually consider expectations to solve the minimum total cost or maximum profit problem [3]. In addition, some researchers have considered different objectives, e.g., maximizing the probability of attaining a given profit level [4–6]. Moreover, some scholars have incorporated marketing factors into the classical newsboy problem, like the effects of advertising with different objectives [7], and returned sales [8,9]. With reference to Khouja and Robbins’ model [7], the demand for single-period products is influenced by the expenditure on advertising and is formulated as a concave function, $E(x_B) = \mu_0 + \mu_B B^\alpha$, where $\mu_0$ and $B$ denote the mean of demand without advertising and the advertising expenditure, respectively, with the effectiveness parameters of $\alpha$.

From the perspective of supply chains, recent studies have been devoted to the newsvendor problem, which is treated as a kind of single-period inventory problem, exploring the random yield problem for some particular production systems [10], investigating the influence of decision makers’ behavior to the risk on the decisions of order quantity [11,12], evaluating the manufacturer’s buyback policies for sharing the inventory risk with the retailer so as to increase order quantity [13], and examining the impact of financial constraint on the supply chain modes [14]. Nevertheless, only one kind of product is considered in these studies.

In fact, a business unit might sell several newsboy-style products at the same time under various constraints. Hadley and Whitin [1] described the multiple-item single-constraint
newsboy problem and solved the minimum total expected cost. Silver et al. [2] developed the solution procedure for maximizing the total expected profit under the multiple-item single-constraint limitation. Lau and Lau [15] extended Hadley and Whitin’s model [1] to consider the multiple-item multiple-constraint newsboy problem, although they indicated that the problem may be impossible to solve. Considering the case of tight constraints, Lau and Lau [16] proposed a workable flow for solving multiple-item multiple-constraint newsboy problems. Subsequently, Abdel-Malek et al. [17], as well as Abdel-Malek and Montanari [18], analyzed this problem and provided an efficient method to solve it. In addition to considering the demand as a probabilistic random variable, Kao and Hsu [19] stressed the uncertainty of demand as a fuzzy variable. Moreover, Shao and Ji [20] explored the fuzzy-demand newsboy problem under a budget constraint.

In real world cases, a reservation policy (also known as an advance-purchase policy) is usually adopted by business units in some industries and for certain commodities, including in the airline [21] and service industries [22], and for newsboy-type products [23,24], since it can reduce the demand uncertainty so as to increase profits. Dana [21] pointed out that, in the airline industry, consumers with particular characteristics usually have an incentive to buy in advance, and also suggested that firms may use an advance-purchase policy to change the allocation of resources so as to increase both business profit and social welfare. Shugan and Xie [22] indicated that, in the service industry, consumers often make advance purchases, and thus separating purchase and consumption can help service providers to generate more profit.

A consumer is often motivated to make a reservation for psychological and economic reasons. Specifically, considering the latter, a discount could be an important incentive for a customer. In general, a higher discount provokes greater customer willingness to make a reservation. In other words, the discount rates have an influence on the sale quantity, and therefore the order quantity. However, a higher discount rate could lead to lower profits being earned by the business unit, or even cause a loss, despite the higher sales. As a result, the determination of the optimal order quantity as well as the optimal discount rate will influence the expected profit arising from the solution to the problem. Consequently, it is worth exploring the newsboy problem with the addition of a reservation policy. Weatherford and Pfeifer [23] examined the system of advance booking of orders, and indicated that the optimal discount rate is an important factor to maximize the expected profit. Chen and Chen [24] solved the newsboy problem considering reservations, and introduced a willingness function with respect to the discount rate. However, the abovementioned studies only consider single newsboy-style products, and have an assumption that the discount rate for reservation or advance-purchase cannot enlarge the original market scale. That is, the demand distribution is not changed. However, more realistically, the extra-demand caused by the discount rate could come from individuals outside the original group of potential customers. For dealing with the above problems, this paper proposes a general algorithm, namely the MCR algorithm, by considering multiple items and a budget constraint in the newsboy problem with a reservation policy. Moreover, the proposed algorithm can be degenerated to solve various kinds of newsboy problems.

The remainder of the paper is organized as follows. In the next section, we will briefly introduce the classical multiple-item budget-constraint newsboy model and its solution procedure. Section 3 then develops the multiple-item budget-constraint newsboy model considering the reservation policy. A resolution algorithm is provided, and this section also introduces the willingness functions, since they play an important role in this study. Next, a numerical example is given to compare the proposed model with the classical one in various situations. A sensitivity analysis of a single-item problem with different budget constraints is also carried out. Finally, concluding remarks are provided in Section 5.

2. CMC model

Silver et al. [2] provided the classical multiple-item budget-constraint newsboy model (the CMC model). Suppose there are n different products. The total profit function \( Z_i^c \) of the ith product in the classical newsboy model depends on the demand and order quantities, and is formulated as

\[
Z_i^c = \left\{ \begin{array}{ll}
(s_i - v_i)x_i + (v_i - c_i)Q_i^c, & x_i < Q_i^c \\
(s_i + p_i - c_i)Q_i^c - px_i, & x_i \geq Q_i^c
\end{array} \right.
\]

where \( i \) is the product index, \( i = 1, \ldots, n \), \( c_i \) is the unit cost of product \( i \), \( s_i \) is the unit selling price of product \( i \), \( v_i \) is the unit salvage value \( (v_i < s_i) \) of product \( i \), \( p_i \) is the unit shortage cost of product \( i \), \( x_i \) is the demand quantity during a period, which is a random variable following a probability distribution of product \( i \), and \( Q_i^c \) is the order quantity (decision variable) of product \( i \).

Since the demand quantity of each product is a random variable following a specific probability distribution, the expected profit is

\[
E(Z_i) = \int_0^{Q_i^c} (s_i - v_i)x + (v_i - c_i)Q_i^c f_x(x) dx + \int_{Q_i^c}^{\infty} (s_i + p_i - c_i)x - px f_x(x) dx
\]

\[
= (s_i - v_i)x + (v_i - c_i)Q_i^c - (s_i + p_i - v_i) \int_{Q_i^c}^{\infty} f_x(x) dx
\]

(2)

Moreover, the total expected profit of all products is the sum of each product’s expected profit as

\[
E(Z_c) = \sum_{i=1}^{n} E(Z_i^c),
\]

subject to

\[
\sum_{i=1}^{n} c_i Q_i^c \leq B,
\]

(4)

where \( B \) is the budget for purchasing products. The Lagrange approach is used to select the multiplier \( \lambda \) and \( Q_i^c \) to maximize

\[
L = \sum_{i=1}^{n} E(Z_i) - \lambda \left( \sum_{i=1}^{n} c_i Q_i^c - B \right).
\]

(5)

For first-order condition, set the partial derivative with respect to \( Q_i^c \) as zero, and obtain \( Q_i^c \), as in the following equation:

\[
Q_i^c = F_x^{-1}[\lambda(s_i + p_i - (1 + \lambda)c_i)/(s_i + p_i - v_i)].
\]

(6)

A given value of \( \lambda \) can determine the corresponding value of \( Q_i^c \), since \( Q_i^c \) is the function of \( \lambda \) and therefore the value of \( \sum c_i Q_i^c \) is obtained. In order to resolve this problem, the unconstrained optimum value of \( Q_i^c \) is usually acquired first. If the \( \sum c_i Q_i^c \leq B \) holds, the \( Q_i^c \) for each product should be adopted. Otherwise, the following algorithm is used to obtain an approximate optimum order quantity of each product.

**CMC algorithm:**

1. **Step 1.** Select an initial positive value of the multiplier \( \lambda \).
2. **Step 2.** Determine each \( Q_i^c \) by

\[
Q_i^c = F_x^{-1}[\lambda(s_i + p_i - (1 + \lambda)c_i)/(s_i + p_i - v_i)].
\]

3. **Step 3.** Compare \( \sum c_i Q_i^c \) with \( B \).
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