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The effect of advertising on the distribution-free newsboy problem

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ABSTRACT

Advertising is very important for the newsboy problem because the shelf-life of the newsboy product is short and advertising may increase sales to avoid overstocking. In this paper, models to study the effect of advertising are developed for the distribution-free newsboy problem where only the mean and variance of the demand are known. As in Khouja and Robbins (2003), it is assumed that the mean demand is an increasing and concave function of advertising expenditure. Three cases are considered: (1) demand has constant variance, (2) demand has constant coefficient of variation, and (3) demand has an increasing coefficient of variation. This paper provides closed-form solutions or steps to solve the problem. Numerical results of the model are also compared with those from other papers. The effects of model parameters on optimal expenditure on advertising, optimal order quantity, and the lower bound on expected profit are derived or discussed.

1. Introduction

The newsboy problem is that of finding a product's order quantity that maximizes the expected profit under probabilistic demand. Researchers' interest in the newsboy problem has increased in recent decades. Khouja (1999) classified the newsboy problem literature into eleven categories and provided useful suggestions for future research. The aim in the classical newsboy problem is maximizing the expected profit. However, Ozler et al. (2009) proposed the multi-product newsboy problem under a Value at Risk (VaR) constraint. A mathematical programming approach was used to find the solution. Wang (2010) studied a game setting where multiple newsvendors with loss aversion preferences are competing for inventory from a risk-neutral supplier. Moreover, the classical newsboy problem assumes that demand follows a specific distribution with the known parameters. However, several authors have analyzed the distribution-free newsboy problem in which only the first two moments of demand, mean and variance, are assumed to be known. Scarf (1958) first addressed the distribution-free newsboy problem and derived a closed-form expression for the optimal ordering rule that maximizes the expected profit against the worst possible distribution of the demand with the mean μ and the variance σ^2. Gallego and Moon (1993) proved the optimality of Scarf's rule and extended Scarf's ideas to four cases. Alfares and Elmorra (2005) extended the models proposed in Gallego and Moon (1993) to incorporate a shortage penalty cost beyond the lost profit. Moon and Choi (1995) derived an ordering rule for the distribution-free newsboy problem with balking. Balking means that once the inventory level falls to the balking level or lower, the per-unit probability of a sale declines from one to less than one. Mostard et al. (2005) studied the case that the returned goods arriving before the end of the selling season can be resold if there is sufficient demand. They found that the distribution-free ordering rule performs well when the coefficient of variation is less than 0.5. Yue et al. (2006) defined the expected value of distribution information, *EVDI*, as the difference in cost functions between a distribution-free decision and the optimal decision under the true demand distribution. The maximum *EVDI* can be used as a robustness measurement for the distribution-free decision.

Advertising is one of the major tools used by companies to target buyers and populations. It consists of impersonal forms of communication conducted through paid media under clear sponsorship. Advertising can be used to build up a long-term image for a product or to trigger quick sales (Kotler, 2001). Vidale and Wolfe (1957) published one of the earliest advertising response models. The model is based on three parameters: a sales decay constant, the saturation level, and the response constant.

The response function may be S-shaped or concave. The S-shaped function indicates first increase in returns and then, after an inflection point, decrease in returns. The concave response function indicates that as advertising expenditures increase, so do sales, but at monotonically diminishing returns from the beginning. Rao and Miller (1975) illustrated a procedure for estimating an S-shaped sales response function from historical data. The procedure uses, as building blocks, distributed lag models that relate market share to advertising expenditures on a market-by-market basis. Little (1979) suggested that several phenomena

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should be considered in building dynamic models of advertising response. These include sales which respond upward and downward at different rates, a steady-state response that can be concave or S-shaped and can have positive sales at zero advertising, the effect on sales of competitive advertising, and advertising-dollar effectiveness that can change over time. Simon and Arndt (1980) reviewed over 100 studies. Evidence consistently indicates that only the concave downward function exists. However, Mahajan and Muller (1986) presented an analytical model that can be used to evaluate the impact of various pulsed and uniform advertising policies. They found that pulsing is optimal only with an S-shaped response function. Using a modified Lanchester model, Mesak and Darrat (1993) demonstrated that the policy of constant advertising spending is superior to a cyclic counterpart, provided that the response functions of the competing firms are concave. Vakratsas et al. (2004) formulated a switching regression model with two regimes, in only one of which advertising is effective. They also estimated a variety of model types, including both standard concave and S-shaped response functions. A sequence of comparisons among these models strongly suggested that the threshold effect exists. However, the debate about the shape of the function continues in the advertising literature.

Khouja and Robbins (2003) assumed that mean demand is increasing and concave in advertising expenditure and studied three cases of demand variation as a function of advertising expenditure: (1) demand has constant variance, (2) demand has a constant coefficient of variation, and (3) demand has an increasing coefficient of variation. They provided closed- or near-closed-form solutions of this problem under several different demand distributions. The present paper extends some of their concepts to the distribution-free newsboy problem.

The rest of this paper is organized as follows. In Section 2, the models are developed. Section 3 illustrates the results of these models using numerical examples and provides managerial insights. Section 4 presents the main findings of this research and points out the direction of future research.

2. The model

We introduce the following notations:

- $c>0$ unit cost,
- $p=(1+m)c>c$ unit selling price, with m being the markup rate,
- $s=(1-d)c<c$ unit salvage value, with d being the discount rate,
- $l=kc$ unit shortage penalty cost, with k being the shortage penalty rate,
- D random demand with mean μ and variance σ^2, $\sigma<\mu$,
- μ_0 expected demand without advertising (the original market size),
- σ_0 standard deviation of the demand without advertising, $\sigma_0<\mu_0$,
- Q order quantity,
- B expenditure on advertising,
- $x^+=\max\{x, 0\}$ the positive part of x.

Consider a distribution-free newsboy problem. Let D be a random variable representing the demand in the period under consideration, with distribution G. As in Gallego and Moon (1993) and Alfares and Elmorra (2005), no assumptions are made on G other than to say that it belongs to the class ϑ of cumulative distribution functions with known mean μ and variance σ^2. In each period, the decision-maker needs to decide the expenditure on advertising B and the order quantity Q to maximize the expected profit against the worst possible distribution of demand.

2.1. Constant variance case (CVC)

Because of diminishing returns from advertising, assume that the mean μ is a concave increasing function of B, i.e., $\mu=\mu_0+\mu_0\omega B^{\alpha}$, where ω and α are empirically determined positive constants that represent the effectiveness of advertising and $0<\alpha<1$. For any $\omega>0$, the larger the values of α and ω, the more effective is the advertising. In the CVC, it is assumed that advertising shifts the mean, but preserves the standard deviation, i.e., $\sigma=\sigma_0$. This situation may occur when a company steps up its advertising efforts to attract more buyers but its competitors remain unaware or have no resources to do the same. The decision-maker would like to maximize the expected profit $\pi^G(Q, B)$, where

$$\max_{Q,B\geq 0}\pi^G(Q,B)=pE(\min\{Q,D\})+sE(Q-D)^+-cQ-lE(D-Q)^+-B,$$

such that $\mu=\mu_0+\mu_0\omega B^{\alpha}$,

$$\sigma=\sigma_0. \tag{1}$$

Note that if $\omega=0$ so that demand is not affected by advertising, the decision-maker will set the expenditure on advertising B to be zero, and (1) reduces to the model presented in Alfares and Elmorra (2005). Next, with the definitions of m, d, k, and the following relationships:

$\min\{Q, D\}=D-(D-Q)^+$ and $(Q-D)^+=(Q-D)+(D-Q)^+$,

the expected profit $\pi^G(Q, B)$ can be rewritten as

$$\pi^G(Q,B)=c\{(d+m)\mu-dQ-(d+m+k)E(D-Q)^+\}-B. \tag{2}$$

Then the following lemmas are required:

Lemma 1.

$$E(D-Q)^+\leq\frac{[\sigma^2+(Q-\mu)^2]^{1/2}-(Q-\mu)}{2}. \tag{3}$$

Lemma 2. *For every Q, there exists a distribution $G^*\in\vartheta$ where the bound* (3) *is tight.*

For the proofs of Lemmas 1 and 2, please refer to Gallego and Moon (1993). Combining (2) and (3) we get

$$\pi^G(Q,B)\geq c\left\{(d+m)\mu-dQ-(d+m+k)\frac{[\sigma^2+(Q-\mu)^2]^{1/2}-(Q-\mu)}{2}\right\}-B. \tag{4}$$

The lower bound is maximized to derive the optimal values of Q and B. Rearranging (4) yields the lower bound of expected profit:

$$\pi_L^G=\frac{c}{2}\left\{(m+d-k)\mu-(d-m-k)Q-(d+m+k)[\sigma^2+(Q-\mu)^2]^{1/2}\right\}-B. \tag{5}$$

From (5), its first partial derivative with respect to Q is

$$\frac{\partial\pi_L^G}{\partial Q}=\frac{-c}{2}\left\{(d-m-k)+\frac{(d+m+k)(Q-\mu)}{[\sigma^2+(Q-\mu)^2]^{1/2}}\right\}. \tag{6}$$

By setting (6) to zero and solving for Q, Scarf's ordering rule is obtained:

$$Q^S=\mu+\frac{\sigma(m+k-d)}{2(kd+md)^{1/2}}=\mu+\frac{\sigma}{2}\left\{\left[\frac{k+m}{d}\right]^{1/2}-\left[\frac{d}{k+m}\right]^{1/2}\right\}. \tag{7}$$

Note that (7) is the same as the result presented by Alfares and Elmorra (2005). Substituting the constraints of (1) into (7), the result is the optimal order quantity:

$$Q_1^*=\mu_0+\mu_0\omega B^{\alpha}+\frac{\sigma_0}{2}\left\{\left[\frac{k+m}{d}\right]^{1/2}-\left[\frac{d}{k+m}\right]^{1/2}\right\}. \tag{8}$$