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A competitive multiple-product newsboy problem with partial product substitution

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ABSTRACT

This paper studies a multi-product competitive newsboy problem with shortage penalty cost and partial product substitution. We characterize the unique Nash equilibrium of the competitive model and analyze some properties of the equilibrium. An iterative algorithm is developed on the basis of approximating the effective demand as well as the expected profit function for each product. Numerical experiments are conducted to illustrate the impacts of product substitution, demand correlation and demand variation on the optimal order quantities and the corresponding expected profits, and to compare the total optimal inventory level of the competitive case with that of the centralized case. The conclusion that competition always results in a higher total inventory level, even under the effect of product substitution is drawn in the symmetric case.

1. Introduction

The classical newsboy problem, which aims at determining the optimal order quantity of a product with stochastic demand to maximize its expected profit in a single period, has been extensively studied in the past few decades. A survey of the literature on the newsboy-type inventory problem and its extensions up to 1999 can be seen in Khouja [1], where 11 categories of extensions are introduced. After that, a number of papers have been devoted to investigating the newsboy-type inventory problem with different considerations, such as Casimir [2], Choi et al. [3], Arcelus et al. [4], Ding and Chen [5], Wang and Webster [6], Wu et al. [7], Keren [8], Kevork [9], Chen and Chen [10], Yang et al. [11] and Feng et al. [12].

In the newsboy-type inventory problem, it is always assumed that in case of a shortage, unsatisfied demand is lost. However, in practice, products with similar colors, styles, sizes or functions can substitute for each other. In view of this, inventory models with product substitution have been considered in a variety of contexts, which can be broadly classified into two types: firm-driven substitution and customer-driven substitution.

Firm-driven substitution, which is called one-way substitution in some research papers, usually assumes that products are sorted into different grades by certain attributes and that only products with higher grades can substitute for those with lower grades (see, e.g., Bitran and Dasu [13], Bassok et al. [14], Hsu and Bassok [15], Hsu et al. [16], Dutta and Chakraborty [17] and Rao et al. [18]). Firm-driven substitution means that the supplier makes decisions for customers on choosing substitutes, whereas customer-driven substitution means that customers can choose substitutes for their first choice product by themselves (Lang [19]). Recently, customer-driven substitution is more prevalent in newsboy-type inventory models.

The research on customer-driven substitution in the literature follows two streams. In the first stream, customer arrival follows a stochastic process and customers make purchasing decisions based on a utility/profit maximization criterion (see, e.g., Smith and Agrawal [20], Mahajan and van Ryzin [21] and Hopp and Xu [22]). In the second stream, each product can substitute for other products with certain probabilities and the cumulative effect of these substitutions on the total demand for a product is evaluated and considered in determining its optimal inventory level. Research papers in this stream can be categorized as either the two-product case or multi-product case, the centralized or competitive version, and with partial or full substitution.

Analysis of the single period newsboy problem with two substitutable products appears in McGillivray and Silver [23], Parlar and Goyal [24], Parlar [25], Pasternack and Drezner [26], Khouja et al. [27] and Nagarajan and Rajagopalan [28]. McGillivray and Silver [23] developed simulation and heuristics to determine the optimal order quantities for the two-product case with identical cost parameters. Parlar and Goyal [24] studied a centralized two-product inventory model with partial substitution. They showed that the total expected profit function is concave for a wide range of parameter settings. Pasternack and Drezner [26] considered the same centralized two-product case but with full substitution. They proved that the total expected profit function is also concave and derived the analytical expressions of the optimal inventory levels.

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Khouja et al. [27] proposed a Monte Carlo simulation to solve the model similar to the one studied by Parlar and Goyal [24]. Using a game-theoretic approach, Parlar [25] obtained the analytical equilibrium solution to the competitive two-product case with partial substitution and proved the uniqueness of the equilibrium. Nagarajan and Rajagopalan [28] solved the centralized newsboy-type inventory problem with two substitutable products whose demands are negatively correlated.

The research on the single period newsboy problem with multiple substitutable products has intensified in recent years. Lippman and McCardle [29] studied a competitive newsboy problem with a random aggregate demand which is allocated first and then reallocated among substitutable products by splitting rules. Netessine and Rudi [30] obtained optimality conditions for both competitive and centralized versions of the single period multi-product inventory problem with partial substitution. However, their analytical results cannot be implemented to search for the optimal solutions directly unless the distribution of the effective demand for each product is available. Based on approximating the effective demand for each product, Rajaram and Tang [31] developed a service rate heuristic to solve the centralized multi-product newsboy problem with normally distributed demand. Zhao and Atkins [32] considered the competitive newsboy model with price competition and partial product substitution. They obtained the unique Nash equilibrium of the model and compared the equilibrium solution with the optimal solution to the classical newsboy problem.

In this paper we employ the service-rate approximation approach used in Rajaram and Tang [31] and Hopp and Xu [22] to estimate the effective demand for each product and develop an iterative algorithm to solve the multi-product competitive newsboy problem with partial substitution which is similar to the competitive model studied by Netessine and Rudi [30]. Based on the iterative algorithm, we address the following questions in computational studies: (1) how does the level of demand variation and demand correlation, and the degree of product substitution affect the optimal order quantities and the corresponding expected profits, respectively? (2) How does the total optimal inventory level change under competition?

While this paper is closely related to the works of Lippman and McCardle [29], Netessine and Rudi [30] and Parlar [25], it differs from these earlier models in several aspects. First, we consider the shortage penalty cost in the expected profit function which is ignored by Lippman and McCardle [29] and Netessine and Rudi [30]. Second, we propose an iterative algorithm to calculate the optimal order quantity of each product. Third, we examine the impacts of product substitution, demand correlation and demand variation on the optimal order quantities and the corresponding expected profits. Finally, we make the comparison of the total optimal inventory levels in the competitive model and the centralized model.

The rest of the paper is arranged as follows. In Section 2, we present the single-period multi-product competitive inventory model. In Section 3, we establish a unique pure-strategy Nash equilibrium and analyze some properties of the equilibrium. In Section 4, we develop an iterative solution algorithm to solve the model and conduct computational studies to illustrate the impacts of product substitution, demand correlation and demand variation on the optimal order quantities and the corresponding expected profits. We also compare the total optimal inventory level of the competitive case with that of the centralized case in this section. In Section 5 we conclude this paper.

2. The model

Consider a competitive version of the single-period (newsboy-type) inventory model. There are n substitutable products in the market, and each product is held by a certain decision-maker (newsboy). At the beginning of the period, each newsboy sends an order to the external supplier. It is assumed that the inventory of each product is replenished in a short time. Then an initial demand for each product, which follows a known continuous random distribution, originates from the stochastically arriving customers. Customers arrive in the market and wish to purchase a certain kind of product. If their first choice product is not available, some customers may attempt to find a substitute. The excess demand for a product transfers to other products based on deterministic fractions. All the customers will leave the market if they suffer a second stockout. The shortage penalty costs are charged for the lost sales. At the end of the period, leftover inventory is salvaged. In the competitive newsboy model employed in this paper, we assume that each newsboy has perfect information about the ordering decisions of other newsboys.

2.1. Notations and assumptions

Let i, j be the product indices, $i, j\in(1, 2,...,n)$. The following notations are defined:

r_i: sales price of product i per unit.
c_i: purchasing cost of product i per unit.
s_i: salvage value of product i per unit.
p_i: shortage penalty cost of product i per unit.
$\boldsymbol{X}_i$: random demand for product i with probability density function $f_{Xi}(x_i)$ and cumulative distribution function $F_{Xi}(x_i)$.
ρ_{ij}: coefficient of correlation between $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$.
α_{ij}: proportion of product i's demand which will switch to product j's inventory when product i is sold out.
Q_i: order quantity of product i.
π_i : expected profit of product i.

In our model, it is assumed that the shortage penalty cost of a product is charged at the same price no matter whether the product serves as a first choice or a substitute in satisfying customers' demand; that is, each newsboy has to pay penalties for any unmet demand for his product before unsatisfied customers leave the market.

In addition, the following assumptions should hold:

Assumption 1. $0\leq a_{ij}\leq 1$ for $i, j=1, 2,...,n$, $i\neq j$.

Assumption 2. $a_{ii}=0$ for $i=1, 2,...,n$.

Assumption 3. $0\leq \sum_{j=1}^{n} a_{ij}\leq 1$ for $i=1, 2,..., n$.

Assumption 4. $r_i>c_i>s_i$ for $i=1, 2,...,n$.

Assumptions 1 and 2 state that a_{ij} is a nonnegative fraction between 0 and 1 which represents the proportion of the excess demand for product i switching to product j under substitution. Assumption 3 states that the total proportion of the excess demand for product i switching to other products should be no larger than 1; that is, not every unsatisfied customer is willing to accept a substitute. Assumption 4 is the common assumption in newsboy-type inventory models.

2.2. The effective demand

In our model, if there is surplus inventory of product i and excess demand (unsatisfied customers) for product j ($j\neq i$, $j\in\{1,2,\ldots,n\}$), then a_{ji} portion of these unsatisfied customers will attempt to purchase product i. This transfer of excess demand represents the substitution between two products.