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An imperfect production process for time varying demand with inflation and time value of money – An EMQ model

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A R T I C L E I N F O

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A B S T R A C T

The paper deals with an *economic manufacturing quantity* (EMQ) model for time-dependent (quadratic) demand pattern. Every manufacturing sector wants to produce perfect quality items. But in long run process, there may arise different types of difficulties like labor problem, machinery capabilities problems, etc., due to that the machinery systems shift from *in-control* state to *out-of-control* state as a result the manufacturing systems produce imperfect quality items. The imperfect items are reworked at a cost to become the perfect one. The rework cost may be reduced by improvements in product reliability i.e., the production process depend on time and also the reliability parameter. We want to determine the optimal product reliability and production rate that achieves the biggest total integrated profit for an imperfect manufacturing process using Euler–Lagrange theory to build up the necessary and sufficient conditions for optimality of the dynamic variables. Finally, a numerical example is discussed to test the model which is illustrated graphically also.

1. Introduction

Most of the classical EMQ models were considered with the realistic assumption of constant demand for the whole year, but in real life, the assumption is not true in general. The demand may vary with time. Another assumption is that the items produced by any manufacturing company are all perfect. But in real life situation, it is not true in general i.e., when the production system is going through a long-run process due to high demand of the produced items which may vary time to time, the manufacturing system shifts from *in-control* to *out-of-control* state, and then the manufacturing system produces perfect as well as imperfect (defective) quality items due to different types of machinery problems, labor problems, etc. A defective cost is allowed to make the defective items as new as the perfect one. Keeping in mind the above facts, we develop a production-inventory model for an imperfect production system considering the variable reliability parameter with effect of inflation and time value of money. The reliability parameter has an impact on the integrated cost function since no manufacturing company can consider that every machinery systems are reliable for life-time. For more accuracy, we consider the unit production cost and the development cost are the functions of reliability parameter which may vary by changes in technology and resources, etc. The integrated profit function is maximized by Euler–Lagrange's method. The major contribution of this model is the variable reliability parameter with time dependent quadratic demand along with the effect of inflation and time value of money. The paper is designed as follows: Introduction are given in Section 1. Literature survey is described in Section 2. Problem Definition, Notations and Assumptions are provided in Section 3. The model is formulated in Section 4. A numerical example is given to test the model in Section 5. Finally, conclusions are given in Section 6.

2. Literature survey

Research on the EMQ models with imperfect production process is being carried out since twenty years. On production-inventory models with unreliable machines have been considered in so many papers till now. Rosenblatt and Lee (1986) found out the effects of process deterioration on the traditional EMQ model with an imperfect production process by considering economic production cycles. Porteus (1986) discussed optimal lot sizing, process quality improvement and set up cost reduction in an imperfect production process. Bielecki and Kumar (1988) found out optimality of zero-inventory policies for unreliable manufacturing systems describing a range of parameter values where zero inventory policies were exactly optimal even when the manufacturing capacity was uncertain. Kim and Huang (1999) extended the model of Rosenblatt

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and Lee (1986), assuming an optimal production run length in deteriorating production process. Further, Chung and Hou (2003) extended the model of Kim and Huang (1999) allowing shortage for the imperfect production process. Giri and Dohi (2005) discussed about the generalization of EMQ model with stochastic machine breakdown and repair in which the time to machine failure, corrective and preventive repair times were all assumed to be random variables. Chen and Lo (2006) developed optimal production run length for items sold with warranty in an imperfect production system where the model was considered with shortages. In their model, an efficient algorithm was developed to find out the optimal solution. Sana, Goyal, and Chaudhuri (2007) discussed the EMQ model with reduction in selling price in an imperfect production process. The demand rate of the defective items without reworking is a non-linear function of reduction rate. Jaber and Bonney (2007) discussed the EMQ model with lot-size dependent learning and forgetting rates where the investigations with the effect of lot-size dependent learning and forgetting rates on the lot-size problem by incorporating the dual-phase learning forgetting model were generalized. Lin, Chiu, and Ting (2008) developed a model on optimal replenishment policy for imperfect quality EMQ model with rework and backlogging. Recently, Chiu, Chen, Cheng, and Wu (2010) generalized the optimization of the finite production rate model with scrap, rework and stochastic machine breakdown. Sarkar, Sana, and Chaudhuri (2010a) extended the EMQ model of Giri and Dohi (2005) with safety stock and reliability parameter where they considered the unit production cost was a function of reliability parameter in imperfect production process. Sarkar, Sana, and Chaudhuri (2010b) extended the EMQ model by introducing optimal reliability, production lot size and safety stock.

After the pioneering attempt by different researchers by considering the constant demand, the general case of a time varying demand rate was first discussed by Silver and Meal (1969). For mathematical convenience, Donaldson's (1977) analytical solution considered finite time horizon and linear time dependent demand. His model's requirements were a substantial amount of computational effort to obtain the optimal time of replenishment. Following Donaldson (1977), significant contribution in this direction came out from researchers like Buchanan (1980), Silver and Peterson (1985), Goyal, Kusy, and Soni (1986), etc. All these models were developed on the assumption that there was no shortage. Deb and Chaudhuri (1987) were the *first* to introduce shortage into inventory with a linear increasing time-varying demand which is an extension of Donaldson (1977) model with shortage. These models were again extended by many researchers like Goswami and Chaudhuri (1991), Teng and Thompson (1996), Hariga (1996), Giri, Chakraborty, and Chaudhuri (2000) and others. Khanra and Chaudhuri (2003) extended the model with quadratic increasing demand over a finite time horizon, allowing shortages. Sana, Goyal, and Chaudhuri (2004) studied an inventory model with linear trend in demand incorporating shortages. Sana and Chaudhuri (2008) developed a deterministic EOQ model with delay in payments and price discounts offer. Sarkar, Sana, and Chaudhuri (2010c) extended the model of Sana and Chaudhuri (2008) with finite replenishment rate, delay in payments and time dependent demand.

Due to high inflation, the economic situation of most of the country may change rigorously and so, it is not possible to ignore the effect of inflation, because inflation declines sharply the purchasing power of money. Assuming inflationary effects on costs, Buzacott (1975) and Misra (1975) extended the EOQ model simultaneously. Bierman and Thomas (1977) developed an EOQ model under inflation with some discounts rates. Misra (1979) presented a model with different inflation rates for various associated costs. Aggarwal (1981) developed a purchase inventory decision model for inflationary conditions. In this direction, some remarkable works were done by Chandra and Bahner (1985), Ray and Chaudhuri (1997), Chen (1998), Chung and Lin (2001), and Yang (2004). Dey, Mondal, and Maity (2008) discussed a two storage inventory problem with dynamic demand under inflation and time-value of money. Chern, Yang, Teng, and Papachristos (2008) established partial backlogging inventory lot size model under inflation and time-value of money. Sarkar and Moon (2011) extended an EPQ model with inflation in an imperfect production system.

Over a long survey of the literature, we found that time-dependent EMQ models with variable reliability parameter is very rare. As far as authors' knowledge goes nobody considered a time-dependent quadratic demand model with variable reliability parameter under the effect of inflation and time-value of money.

3. Problem Definitions, Notation and Assumptions

Problem Definition: By considering EMQ model, we want to develop a production-inventory model. In any production system, when the machines are gone through a long-run process, there may arise different types of difficulties in production which results the production of defective items. These defective items are reworked at a cost to make the products as new as perfect one remembering the brand image of the manufacturing system. The production of the defective items increases with time and reliability parameter. For this reason, we consider a time-dependent quadratic demand pattern in which the development cost and unit production cost are dependent on the reliability parameter. In spite of that, we consider different types of cost function related with the production system incorporating with the inflation and time value of money. We want to find the associated profit function which we have to maximized by the control theory.

The following notation and assumptions are used to develop the model:

Notation:

D – quadratic demand at time $t, t \geqslant 0$ and $D = a + bt + ct^2$;
$Q(t)$ – on-hand inventory at time $t \geqslant 0$;
$\dot{Q}$ – derivative of $Q(t)$ with respect to time t;
C_0 – constant material cost per unit item;
$C_1(\eta)$ – development cost dependent on reliability parameter;
$C_P(\eta, t)$ – production cost of unit item;
C_h – holding cost per unit time;
C_d – rework cost per defective item in the imperfect production system;
p – selling price per unit item sold;
α – variation constant of tool/die costs;
$S(t)$ – production rate at time t;
η – product reliability parameter;
η_{min} – minimum value of the reliability parameter η;
η_{max} – maximum value of the reliability parameter η;
R – the difficulties in increasing reliability;
M – the fixed cost like labor and energy costs which is independent of reliability factor η;
N – the cost of technology, resource and design complexity for production when $\eta = \eta_{max}$;
$\tau = r - i$ where r = interest per unit currency and i = inflation per unit currency;
T – the length of production-inventory cycle.

Assumptions:

1. The model is developed only for a single item in an imperfect production process for quadratic demand pattern i.e., $D = a + bt + ct^2$ where $a, b, c > 0$.
2. In this production process, two types of items are produced: perfect and imperfect, the perfect quality items are ready for