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Parametric analysis of setup cost in the economic lot-sizing model without speculative motives☆

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Abstract

In this paper we consider the important special case of the economic lot-sizing problem in which there are no speculative motives to hold inventory. We analyze the effects of varying all setup costs by the same amount. This is equivalent to studying the set of optimal production periods when the number of such periods changes. We show that this optimal set changes in a very structured way. This fact is interesting in itself and can be used to develop faster algorithms for such problems as the computation of the stability region and the determination of all efficient solutions of a lot-sizing problem. Furthermore, we generalize two related convexity results which have appeared in the literature.

Keywords: Economic lot-sizing; Parametric analysis; Stability; Efficient solutions; Dynamic programming

1. Introduction

In 1958 Wagner and Whitin published their seminal paper on the "Dynamic Version of the Economic Lot-Size Model", in which they showed how to solve the problem considered by a dynamic programming algorithm. It is well known that the same approach also solves a slightly more general problem to which we will refer as the economic lot-sizing problem (ELS). Recently, considerable improvements have been made with respect to the complexity of solving ELS and some of its special cases (see [1–3]). Similar improvements can also be made for many extensions of ELS (see [4]).

In this paper we consider the important special case of ELS in which there are no speculative motives to hold inventory, i.e., the marginal cost of producing one unit in some period plus the cost of holding it until some future period is at least the marginal production cost in the latter period. For this model we analyze the effects of varying all setup costs by the same amount. This is equivalent to studying the set of optimal production periods when the number of such periods changes. We will show that this optimal set changes in a very structured way. This fact is interesting in itself and can be used to develop faster algorithms for such problems as the computation of the stability region and the determination of all efficient solutions of a lot-sizing problem. Furthermore, we will generalize

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two related convexity results which have appeared in the literature.

The paper is organized as follows. In Section 2 we state several useful results about the economic lot-sizing problem without speculative motives. In Section 3 we perform a parametric analysis of the problem. We will characterize how the optimal solution changes when all setup costs are reduced by the same amount and we will present a linear time algorithm to calculate the minimal reduction for which the change actually occurs. In Section 4 we discuss applications of the results of Section 3. Finally, Section 5 contains some concluding remarks.

2. The economic lot-sizing problem without speculative motives

In the *economic lot-sizing problem* (*ELS*) one is asked to satisfy at minimum cost the known demands for a specific commodity in a number of consecutive periods (the *planning horizon*). It is possible to store units of the commodity to satisfy future demands, but backlogging is not allowed. For every period the production costs consist of two components: a cost per unit produced and a fixed setup cost that is incurred whenever production occurs in the period. In addition to the production costs there are holding costs which are linear in the inventory level at the end of the period. Both the inventory at the beginning and at the end of the planning horizon are assumed to be zero.

We will use the following notation:

T: the length of the planning horizon,
d_i: the demand in period $i \in \{1, \dots, T\}$,
p_i: the unit production cost in period $i \in \{1, \dots, T\}$,
f_i: the setup cost in period $i \in \{1, \dots, T\}$,
h_i: the unit holding cost in period $i \in \{1, \dots, T\}$.

Furthermore, we define $d_{ij} \equiv \sum_{t=i}^{j} d_t$ for all i,j with $1 \leqslant i \leqslant j \leqslant T$.

As shown in [3] an equivalent problem results when all unit holding costs are taken 0, and for all $i \in \{1, \dots, T\}$ the unit production cost p_i is replaced by c_i, defined as

$$c_i \equiv p_i + \sum_{t=i}^{T} h_t.$$

This reformulation can be carried out in linear time and it changes the objective function value of all feasible solutions by the same amount. From now on we will focus on the reformulated problem.

In this paper, we assume that $c_i \geqslant c_{i+1}$ for all $i \in \{1, \dots, T-1\}$. Note that if c_i were less than c_j for some $j > i$, then this could be perceived as an incentive to hold inventory at the end of period i (in order to avoid that the higher unit production cost in period j will have to be paid). Under our assumption on the unit production costs this incentive is not present. Therefore, this special case is known as the *economic lot-sizing problem without speculative motives*. Note that in the model originally considered by Wagner and Whitin [5] it is assumed that $h_i \geqslant 0$ and $p_i = 0$ for all $i \in \{1, \dots, T\}$. Because $c_i = p_i + \sum_{t=i}^{T} h_t$, it is easily seen that this model is an example of a lot-sizing problem without speculative motives.

It is well known that economic lot-sizing problems can be solved using dynamic programming. For problems without speculative motives, the dynamic programming algorithm can be implemented such that it requires only O(T) time (see [1–3]). We will now briefly review such an implementation.

The key observation to obtain a dynamic programming formulation is that it suffices to consider only feasible solutions that have the *zero-inventory property*, i.e., solutions in which the inventory at the beginning of production periods is zero. The latter implies that if i and j are consecutive production periods with $i < j$, then the amount produced in period i equals d_{ij-1}. From now on, we will only consider solutions with this property. Also, note that we may assume that setups only take place in production periods, even if some of the setup costs are zero. Hence, solutions can completely be described by their production periods which coincide with the periods in which the setups occur.

Let the variable $F(i)$, $i \in \{1, \dots, T\}$, denote the value of the optimal production plan for the instance of ELS with the planning horizon truncated after period i, and define $F(0) \equiv 0$. For $i = 1, \dots, T$ the value of $F(i)$ can be calculated using the following forward recursion:

$$F(i) := \min_{0 < t \leqslant i} \{F(t-1) + f_t + c_t d_{ti}\}.$$