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# On optimal two-stage lot sizing and inventory batching policies

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**Abstract**

A recent paper considered the two-stage lot-sizing problem with finite production rates at both stages. The problem was classified according to: whether the production rate at the first stage is greater or less than that at the second stage, whether the production batch size at the first stage is greater or less than that at the second stage and whether the transfer from the first stage to the second stage is continuous or in batches. The objective of this note is to offer an alternative (more direct and intuitive) way to derive and present essentially similar results and also to extend the analysis by relaxing one of the assumptions. © 2000 Elsevier Science B.V. All rights reserved.

*Keywords:* Inventory batching policies; Two-stage lot sizing

## 1. Introduction

In an interesting recent paper [1] Kim carried out a thorough analysis of the two-stage lot-sizing problem with finite production rates at both stages. By considering the various ways of classifying such a model an optimal solution procedure was derived and presented. The objective of this note is to suggest an alternative way of performing the analysis which we believe to be more intuitive and concise and therefore possibly easier to understand. In addition, by relaxing one of the key assumptions it is shown how the policy space can be extended and lower cost policies derived.

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## 2. Definitions and assumptions

For convenience we shall follow most of the definitions and assumptions in [1] which we set out again here. In addition we introduce one or two further terms.

At the first production stage (stage 2) a raw material is manufactured at a finite rate, in batches, to produce an intermediate product which we shall call process stock. Stock is transferred, from stage 2 to 1, either continuously ('continuous transfer') or as soon as a stage 2 batch has been finished ('batch transfer'). We note, in passing, that another possible rule for stock transfer is to transfer to stage 1 whatever process stock is required to manufacture a batch at stage 1 so that this stock arrives just at the time when the manufacture of the stage 1 batch is due to start. This may be applicable if the process stock holding cost increases after the transfer is made (see, for example, [2,3]).


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At stage 1 the process stock undergoes further manufacturing at a finite rate, in batches, to produce a finished good (1 unit of process stock making 1 unit of finished goods stock). Batch set up costs are incurred at both stages and stockholding costs are incurred on both process stock and finished goods stock. There is a constant external demand for finished goods which has to be met. Everything about the system is assumed to be deterministic and all the stock variables are assumed to be continuous in nature.

According to the policy structure considered by Kim production at stage 2 is done in equal-sized batches of $Q_2$ and at stage 1 in equal-sized batches of $Q_1$. Whichever of $Q_1$ or $Q_2$ is the greater determines the basic repeating production cycle. For example, if $Q_2 \geqslant Q_1$ then the repeating cycle (of duration $Q_2/D$) starts when the production of a batch at stage 2 starts and finishes when the production of the next batch at stage 2 starts. Within this cycle there are $k$ batches of finished goods produced. Because $Q_2$ will not in general be an integral multiple of $Q_1$ the actual pattern of finished goods production consists of $(k-1)$ batches of size $Q_1$ and a final batch of size $\delta Q$ $(0 < \delta Q \leqslant Q_1)$, determined by $Q_2 = (k-1)Q_1 + \delta Q$. Similar comments apply if $Q_2 \leqslant Q_1$. Therefore it is not, in general, the case that all batch sizes at a given stage are equal. In this note we consider how the analysis might be extended if all the batch sizes at a given stage are allowed to differ.

### 2.1. Definitions

- $i$ — The stage/inventory index ($i = 2$ for stage 2/process stock and $i = 1$ for stage 1/finished goods stock).
- $D$ — The constant (continuous) demand rate for finished goods (at stage 1).
- $P_i$ — The production rate at stage $i$ ($P_i > D$ for the problem to be non-trivial).
- $Q_i$ — The (target) production batch size at stage $i$.
- $F_i$ — The set up cost at stage $i$.
- $c_i$ — The stockholding cost per unit per unit time at stage $i$ (we assume that $c_2 \leqslant c_1$).
- $\mathrm{AS}_i$ — The average stock level of inventory $i$.
- ATI — The average total inventory in the system ($= \mathrm{AS}_1 + \mathrm{AS}_2$).
- TC — The average total cost per unit time of set up and stockholding.

Two general observations can be made:

(i) For some parameter combinations it will be easier to compute $\mathrm{AS}_1$ and ATI and then deduce $\mathrm{AS}_2$ (as $\mathrm{ATI} - \mathrm{AS}_1$) than to compute $\mathrm{AS}_2$ directly. Similarly, it may sometimes be more convenient to express the total stockholding cost per unit time as $c_2\mathrm{ATI} + (c_1 - c_2)\mathrm{AS}_1$ (rather than $c_1\mathrm{AS}_1 + c_2\mathrm{AS}_2$).

(ii) Since $c_2 \leqslant c_1$ we generally wish to have a policy which, other things being equal, minimises finished goods stock and this will generally be achieved by manufacturing finished goods in equal batch sizes. In this context we will make use of the following fairly well-known result (which can be proved by the method of Lagrange multipliers):
If $K$ is a positive constant then $\sum_{j=1}^{n} a_j^2$ is minimised, subject to the constraint $\sum_{j=1}^{n} a_j = K$, when all the $a_j$ are equal. (A)

### 2.2. The eight parameter settings

Kim used the term 'policy' to describe the eight different structural combinations of model parameters and decision variables under consideration. We shall use the term 'setting' and reserve the term 'policy' to identify a particular specification of lot sizes. The eight settings are:

*Setting 1* $Q_2 \geqslant Q_1, P_2 \geqslant P_1$, batch transfer,
*Setting 2* $Q_2 \leqslant Q_1, P_2 \geqslant P_1$, batch transfer,
*Setting 3* $Q_2 \geqslant Q_1, P_2 \leqslant P_1$, batch transfer,
*Setting 4* $Q_2 \leqslant Q_1, P_2 \leqslant P_1$, batch transfer,
*Setting 5* $Q_2 \geqslant Q_1, P_2 \geqslant P_1$, continuous transfer,
*Setting 6* $Q_2 \leqslant Q_1, P_2 \geqslant P_1$, continuous transfer,
*Setting 7* $Q_2 \geqslant Q_1, P_2 \leqslant P_1$, continuous transfer,
*Setting 8* $Q_2 \leqslant Q_1, P_2 \leqslant P_1$, continuous transfer.

Typical patterns of stock against time are shown in Figs. 1–8. In these figures the finished goods inventory is always shown as a solid line. In addition, the wide dash line shows either the total inventory or the process inventory, whichever is more helpful in a particular setting. The narrow dash