

Multiproduct lot sizing for finite production rate[☆]

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Abstract

This article deals with the situation where the producer manufactures several products with given production rates on a single machine. Only one product can be produced at a time on the machine. As an example, the producer may use higher or lower quality components in the production. Also, the products may have different holding costs. We present a model, where the demand appears at discrete points in time, and all demand must be met. We assume a deterministic, varying demand. By a production batch we mean a sequence of individual demands that is possible to manufacture in a continuous run to satisfy all the demands on time. A production run consists of a sequence of batches. There is a fixed set-up cost associated with each production run. By a production schedule we mean a sequence of “run” intervals such that each of them is partitioned into parts – for each product. This paper is concerned with the problem of how to determine the production schedule that minimizes the total production and holding cost over a finite time horizon. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

We consider the problem of finding the optimal schedule of production runs of single machine to meet all discrete multiproduct demands which are required at discrete points of time. There is a time-invariant fixed joint setup cost. We assume there is no costs associated with switching the production rate. Each product has its own time-proportional stock holding cost and production rate. This type

of finite time horizon problem is related to the multiproduct capacitated lot-sizing problem in which the production rate determines the period's production capacity.

A schedule is described by the collection of production amount functions – each one for an individual product. The sum of these functions is equal to zero on the intervals where no product is manufactured. The sequential segments (where it is positive) determine the sequence of production batches. The objective is to determine the (joint) production amount function that minimizes the total cost over a finite time horizon.

Hill [1], in the case of a single product, shows how the finite production rate problem can be transformed into a discrete time uncapacitated lot-sizing problem of the Wagner–Whitin type. We refer to Hill [1] for a detailed literature review of

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these relations. The objective of the present paper is to generalize the approach suggested by Hill to the multiproduct case. We do it by a natural generalization of his methodology.

We assume that products are similar enough to be aggregated (using the same units). To find the joint batch production time, we decompose it for each product to solve an auxiliary allocation problem which has received considerable attention also in our paper [2]. Batching is clustering the products for manufacturing processing in the same production run. In this way, batching typically generates cycle inventory which can be advantageous for economic reasons; see Kuik and Salomon [3] for a discussion of batching analysis. Production in larger quantities reduces the number of setups.

2. The problem description

The following terminology is used:

- n the number of products,
- t_k the time at which the k th demand as a multiproduct vector occurs, $k = 1, \dots, N$,
- d_k^i the quantity demanded from the i th product in the k th demand,
- (d_k^1, \dots, d_k^n) the vector of quantities of required products in the k th demand; we call it multiproduct-demand (or, simply, demand),
- P_i the finite production rate of the i th product,
- c the fixed joint setup cost,
- h_i the stockholding cost per unit per unit time for product i .

We have the planning horizon t_N . It is needed to indicate all subintervals of $[0, t_N]$ for producing i th product for $i = 1, \dots, n$.

Let $\{[a_v^i, b_v^i] \subset [0, t_N] | i = 1, \dots, n; v = 1, \dots, M_i; M_i < \infty\}$ be a family of intervals such that

- (1) every two different intervals have disjoint interiors, and
- (2) there are no i and $v \neq v'$ such that $b_v^i = a_{v'}^i$.

The intervals $[a_v^i, b_v^i]$, $v = 1, \dots, M_i$, denote the time intervals at which the i th product is manufactured. Observe that condition (1) means that only one product, if any, can be produced at a time. Condition (2) implies that each point a_v^i is either a *switching point* (if there is a point $b_{v'}^i$ such that $a_v^i = b_{v'}^i$) or a *setup point*, otherwise. The family of intervals satisfying (1) and (2) is called a *production schedule*. The production amount, corresponding to a production schedule, of the i th product at the moment t can be expressed as

$$P^i(t) = P_i \sum_{\{j|b_j^i \leq t\}} P_i(b_j^i - a_v^i) + P_i \delta_i(t),$$

where

$$\delta_i(t) = \begin{cases} t - a_v^i & \text{if } a_v^i < t < b_v^i, \\ 0 & \text{otherwise.} \end{cases}$$

Each *production amount function* P^i (as well as the sum of all functions P^i) is a continuous peicewise linear function with the coefficient P_i on each interval $[a_v^i, b_v^i]$, $v = 1, \dots, M_i$, and with the coefficient 0, otherwise. The family $\{P^1, \dots, P^n\}$ is also called a production schedule.

We say that a production schedule is *feasible*, if and only if, for each product $i = 1, \dots, n$,

$$P^i(t) \geq \sum_{\{k|t_k \leq t\}} d_k^i, \quad t \in [0, t_N].$$

The objective is to find a feasible production schedule which minimizes the sum of setup and holding costs:

$$c \sum_{i=1}^n \sum_{v=1}^{M_i} \sigma(a_v^i) + \sum_{i=1}^n h_i \int_0^{t_N} \left[P^i(t) - \sum_{\{k|t_k \leq t\}} d_k^i \right] dt,$$

where

$$\sigma(a_v^i) = \begin{cases} 1 & \text{if } a_v^i \text{ is a setup point,} \\ 0 & \text{otherwise.} \end{cases}$$

Observe that for every production schedule we can establish the *run production intervals*. Namely, for each setup point T , at which the production starts, there exists exactly one point $T^* > T$ at which the production finishes, i.e. $T^* = b_v^i$ for some v, i and it is not a switching point.

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