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Economic lot sizing with learning and continuous time discounting: Is it significant?

Mohamad Y. Jaber[a,*], Maurice Bonney[b]

[a]School of Industrial Engineering, Department of Mechanical Engineering, Ryerson Polytechnic University, Toronto, ON, Canada M5B 2K3

[b]School of Mechanical, Materials, Manufacturing Engineering and Management, University of Nottingham, University Park, Nottingham NG7 2RD, UK

Abstract

Learning curves are a means of representing continuous improvement in firms. Such improvements bring savings in production costs. This may also allow smaller batches to be produced more frequently and hence bring further savings in holding costs. Earlier research advocated that for more realistic modelling of inventory problems, the holding cost should be evaluated means of the internal discount rate of the firm (C. Van Deft, J.P. Vail, International Journal of Production Economics 44 (1996) 255–265). This paper examines whether, when learning is considered, it is reasonable to ignore the effect of continuous time discounting of costs by investigating the effect of learning and time discounting both on the economic manufacturing quantity and minimum total inventory cost. Numerical examples are provided to illustrate the solution procedure for the mathematical model developed. Although the analysis yields different economic order quantities, the difference in cost from the quantities derived using Wilson lot size formula is not significant.

Keywords: Learning; EMQ/EOQ; Continuous time discounting; Intermittent production

1. Introduction

The use of the learning curve has been receiving increasing attention due to its applications in different operations management areas; e.g. inventory management. In general, learning is an important consideration whenever an operator begins production of a new product, changes to a new machine or restarts production after some delay. This implies that the time/cost needed to produce a product will reduce as the individual, or group of individuals, becomes more proficient.

The economic manufacturing quantity model with learning in production has been treated in Refs. [1–15]. These researches concluded that in the presence of learning in production, the optimal lot size policies were to produce more lots of smaller sizes, and that resulted in substantial savings in total inventory costs. Traditional inventory models [16,17] do not account for the time-value of money. The effects of discounting may be considered when determining the economic manufacturing quantity [18–23].

* Corresponding author. Tel.: 416-979-5000-Ext. 7623; fax: 416-979-5265.

E-mail address: mjaber@ryerson.ca (M.Y. Jaber).

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The basis of this research stems from the principles discussed above. First, learning has economic implications for the design of inventory systems due to the continuous improvement in production capacity. Secondly, as advocated in [18–23], discounting effects should be considered for the realistic modelling of the economic lot size problem. However, none of these Refs. [1–23] studied the combined effect of learning in production and continuous time discounting on the economic manufacturing quantity. That is the focus of this paper.

The rest of the paper is organised as follows: Section 2 presents an introduction to learning curve theory, Section 3 describes the mathematical model that is used, Section 4 discusses the numerical results drawn from the mathematical model developed in Section 3, and finally, Section 5 presents a summary and conclusions.

2. Learning curve theory

Early investigations of learning revealed that the time required to perform a task declined as experience with the task increased. The first attempt made to formulate relations between learning variables in quantitative form, by Wright [24], resulted in the theory of the "learning curve".

Wright's power function formulation of the "Aircraft Learning Curve", known to some as Wright's model of progress, can be represented as

$$T_j = T_1 j^{-b}, \tag{1}$$

where T_j is the time to produce the jth unit, T_1 is the time to produce the first unit (note that the initial production rate is $p_1 = 1/T_1$), j is the production count, and b is the slope of the learning curve when represented on a logarithmic scale. Wright's simple mathematical expression is easy to implement and understand by managers. Fig. 1 illustrates Wright's learning curve.

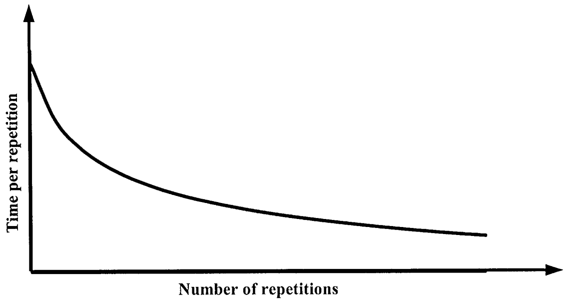

Fig. 1. Wright's learning curve.

3. The mathematical model

Consider an inventory process where an item is produced in batches, at an increasing production rate because of learning, and consumed at a constant rate of r units per time period (e.g., units per year). In any production cycle i, define t_{pi} as the time to produce q_i units and build a maximum inventory of Z_i units, and t_{di} as the time required to deplete Z_i. The level of inventory can then be expressed as a function of time, $\Phi_i(t)$, as

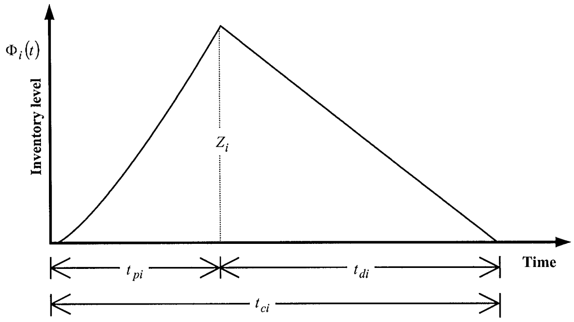

Fig. 2. Variation in inventory level with learning.

$$\Phi_i(t) = q_i(t) - rt \quad \text{for } 0 \leqslant t \leqslant t_{\mathrm{pi}}, \tag{2}$$

$$\Phi_i(t) = -rt + rt_{\mathrm{ci}} \quad \text{for } t_{\mathrm{pi}} \leqslant t \leqslant t_{\mathrm{ci}}, \tag{3}$$

where t_{ci} is the cycle time, i.e. the time for producing and consuming the q_i units, and equals the sum of t_{pi} and t_{di}. Fig. 2 illustrates the hypothesised variation in the inventory level given in Eqs. (2) and (3) over the cycle time t_{ci}. As agreed by many researchers [2,10–12], a good approximation of the