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The economic production lot-sizing problem with imperfect production processes and imperfect maintenance

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Abstract

This paper deals with an integrated model for the joint determination of economic production quantity and preventive maintenance (PM) level for an imperfect process having a general deterioration distribution with increasing hazard rate. The effect of PM activities on the deterioration pattern of the process is modeled using the imperfect maintenance concept. It is assumed that, after each PM, the age of the system is reduced proportional to the PM level. A numerical example is used to illustrate the effect of PM level on the cost of nonconforming items and restoration cost due to shift to the out-of-control state. It is found that performing PM will yield reductions in quality related costs. It is more economical to perform PM when the extra PM cost is compensated for by the reduction in quality related costs.

Keywords: Production lot-sizing; Preventive maintenance; Imperfect processes; Integrated models

1. Introduction

The role of the equipment condition in controlling quality and quantity is well known [3]. Equipment must be maintained in top operating conditions through adequate maintenance programs. Despite the strong link between maintenance production and quality, these main aspects of any manufacturing system are traditionally modeled as separate problems. Few attempts have been made to integrate them in a single model that captures their underlying relationships.

The classical economic production quantity (EPQ) model [9,17] assumes that the output of the production system is defect free. Rosenblatt and Lee [15] have found that, when the production process is subject to a random process deterioration that shifts the system from an in-control state to an out-of-control state (and producing nonconforming items then), the resulting optimal EPQ is smaller than that of the classical model. Porteus [12] has observed similar results. Lee and Rosenblatt [7] also incorporated maintenance by inspection with restoration cost dependent on the detection delay. They assumed that the deterioration of the process is exponentially distributed. Their work was extended by Lin et al. [8] for the case where the deterioration of the process follows a general distribution. Huang et al. considered a further extension using a preventive maintenance (PM) that either brings the system to the 'as good as new' condition or the system will fail

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immediately thereafter because of faulty maintenance. For further review of the determination of EPQ with imperfect production processes, the reader is referred to Ben-Daya and Rahim [5].

The purpose of this paper is to develop a model that links EPQ, quality and maintenance requirements for a process having a general deterioration distribution and where the maintenance level is optimized.

PM activities are known to improve the reliability of equipment. In other words, a preventively maintained equipment will experience less failures. In many PM models, the system is assumed to be 'as good as new' after each PM action. However, a more realistic situation is one in which the failure pattern of a preventively maintained system changes. One way to model this is to assume that, after PM, the failure rate of the system is somewhere between 'as good as new' and 'as bad as old'. This concept is called imperfect maintenance and was introduced by many authors [10,11]. It can be assumed that the failure rate of the equipment is decreased after each PM. This amounts to a reduction in the age of the equipment. In this paper, we assume that the reduction in the age of the equipment is proportional to the cost of PM. This change in the age of the equipment will affect the time to shift to the out-of-control state distribution and consequently the amount of nonconforming items. The maintenance level that produces the least total expected cost corresponds to the optimal PM level.

Numerical examples are used to illustrate the proposed model and study the effect of maintenance level on EPQ and quality costs. In particular, it is observed that, compared to the case with no PM, the extra cost of maintenance results in lower overall expected cost due to the reduction in quality related costs.

The remainder of this paper is organized as follows. In the next section, we define the problem and develop the necessary notation and assumptions. The mathematical model is developed in Section 3. Section 4 presents a description of the solution procedure used along with some illustrative examples. Finally, Section 5 contains a summary of the paper, some concluding remarks and suggestions for future research.

2. Problem definition, notation and assumptions

Consider a production process producing a single item. A production cycle begins with a new system which is assumed to be in an *in-control* state, producing items of acceptable quality. However, after a period of time in production, the process may shift to an *out-of-control* state. The elapsed time for the process to be in the in-control state, before the shift occurs, is a random variable assumed to follow a general distribution with increasing hazard rate. The process is inspected at times $t_1, t_2, \ldots, t_m$ to assess its state and at the same time PM activities are carried out. The production cycle ends either when the system is out of control or after m inspection intervals whichever occurs first. The number m is a decision variable. The process is then restored to the in-control state and to the as good as new condition by maintenance and/or replacement. The usual assumptions of the classical EPQ model apply here. In particular, the demand is constant and continuous and it must be met.

Next, we state the notation and assumptions made to develop the integrated model for the joint determination of EPQ and preventive maintenance level for an imperfect production process:

D	demand rate in units per unit time
P	production rate in units per unit time
T'	expected production cycle for each production lot
T	expected actual production time for each cycle (production run)
K	setup cost excluding maintenance cost
C_h	holding cost per unit time
s	cost incurred by producing a nonconforming item
v	inspection cost
α	percentage of nonconforming units produced when the process is in the out-of-control state
c_{pm}	cost of scheduled maintenance
τ_i	detection delay during interval i