

On the relaxation of multi-level dynamic lot-sizing models[☆]

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Abstract

In this paper the multi level dynamic lot sizing problem is analyzed when setup and holding costs do not depend on time. A zero–one integer programming formulation and its linear relaxation are investigated. The paper shows that for certain classes of problems the relaxed models provide integer solution when the number of periods is below a certain threshold. A counter example shows that at least six periods are required to obtain a unique non-integer solution of the relaxed problem. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Flexibility is one of the oldest themes of production and operations management. Lot sizing, as one of the most important elements of flexibility, has been playing an important role in the literature of production and operations management, especially since reducing setup costs and setup times has become one of the most effective tools. Parallel with this process, the notion of mass production began to decline, and today JIT, lean production, flexible manufacturing are among the main competing manufacturing paradigms. These changes in the manufacturing paradigms have been reflected by the EOQ-connected literature as well. The pioneer works of Porteus [1–3] deal first

time with the problem of setup cost reduction in the EOQ model, however less attention has been paid to the same problem in dynamic case, i.e. when demand is not constant. As demand can rarely be fully smoothed, knowing the required level of setup cost reduction in order to decrease the number of setups during certain time periods, is an important issue for managers. Thus studying the robustness of a given optimal solution, especially with respect to setup cost, and knowing the nature of the problem are useful.

The setup cost stability region (SR) for an optimal solution is defined as the set of all setup cost values for which a given solution remains optimal.

The sensitivity analysis of the single level dynamic lot sizing problem arises first time in the paper of Richter [4] and—following the idea of dynamic programming—requiring cost inputs to have the same regeneration set for every time period, he could reveal a subset of SR. Later Richter and Vörös [5] have analyzed the multi-

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stage lot sizing problem but they could show results only based on the same assumption extended to the multi level problem. They also expressed that they could reveal the whole SR by using a full enumeration which is obviously not an effective way.

The next step in this field was made by Chand and Vörös [6] when they applied a new approach to the problem. For the single level case—with backlogging—they proved that the total cost function is convex in the number of setups. (We mention also that the convex nature of the holding cost function—without backlogging—is also derived in the paper of [7]. Similar results for the multi level dynamic lot-sizing case were published by Vörös [8].

The nature of the multi level dynamic lot sizing problem has attracted many researchers (see for example [9–12]) and the linear relaxation of the zero–one integer programming formulation of the dynamic lot sizing problem plays an important role in the analysis of the stability regions. When the zero–one formulation can be substituted by linear programming models the proof of some theorems required to identify stability regions becomes simple. When setup and holding costs can vary period by period, Pochet and Wolsey [13] defined a two level, four period problem for which the relaxed version has a unique non-integer solution. This paper shows that with time independent setup and holding costs the relaxed version of the integer problem with four and five periods has always integer solution. The paper presents a six period problem that has no optimal integer solution despite the fact that the setup and holding costs are constant through periods.

2. The multi stage serial assembly problem

According to a general description of the serial assembly system [12] the production and inventory system is assumed to have M facilities in series, and the input to facility $(m + 1)$ comes from the production at facility m . Raw materials are available in unlimited amounts as input to facility 1. Facility M produces assemblies which are used to supply the customer demand. All facilities are

allowed to carry inventories while the M th facility carries the finished goods inventory. It is assumed that production and shipments are instantaneous, and that one unit of production on facility $(m + 1)$ requires one unit of input from facility m .

Let D_t denote the demand in period t ; it is assumed that demand is known for periods 1 to T . Let X_{mt} denote the production at facility m in period t , and the cost of this production is $C_{mt}(X_{mt})$. I_{mt} is the inventory at the end of period t at facility m and the corresponding cost is $H_{mt}(I_{mt})$ for all $m \in \langle 1, M \rangle$ and $t \in \langle 1, T \rangle$, where $\langle a, b \rangle = \{a, a + 1, \dots, b\}$. Then the multi stage assembly problem can be formulated as:

$$\text{MIN} \sum_{t=1}^T \sum_{m=1}^M [C_{mt}(X_{mt}) + H_{mt}(I_{mt})], \quad (1a)$$

$$X_{M+1,t} = D_t, \quad t \in \langle 1, T \rangle, \quad (1b)$$

$$I_{m,t-1} + X_{mt} = X_{m+1,t} + I_{mt} \quad \text{for all } m \in \langle 1, M \rangle, \quad t \in \langle 1, T \rangle, \quad (1c)$$

$$I_{m0} = I_{mT} = 0, \quad m \in \langle 1, M \rangle, \quad (1d)$$

$$I_{mt}, X_{mt} \geq 0, \quad m \in \langle 1, M \rangle, \quad t \in \langle 1, T \rangle. \quad (1e)$$

According to Crowston and Wagner [10], problem (1) can be represented as a network flow problem with one source and T sinks (shown by Fig. 1). For concave cost functions C and H respectively, Zangwill [14] has stated that there exists an optimal solution corresponding to an extreme flow, i.e. a node can have at most one positive input (see also the proof of [9] for more general cost functions). This means that $I_{m,t-1}X_{mt} = 0$ for all $m \in \langle 1, M \rangle$ and $t \in \langle 1, T \rangle$ in at least one optimal solution.

Now, let us consider problem (1) with a cost function used widely in the literature:

$$C_{mt}(X_{mt}) = \begin{cases} S_m + c_m X_{mt} & \text{for } X_{mt} > 0, \\ 0 & \text{for } X_{mt} = 0, \end{cases} \quad (2)$$

and

$$H_{mt}(I_{mt}) = h_m I_{mt} \quad \text{for all } m \text{ and } t \text{ with}$$

$$h_m \leq h_{m+1}, \quad m \in \langle 1, M - 1 \rangle,$$

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