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Omega 31 (2003) 539–545
omega
The International Journal of Management Science
www.elsevier.com/locate/dsw

Optimal lot sizing under continuous price decrease

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Received 18 November 2002; accepted 8 August 2003

Abstract

An important characteristic of high-tech industries is decreasing component prices over time. In the personal computer industry, some component prices decline at a rate of 1% per week. This paper develops an inventory model for products experiencing continuous decrease in unit price. We develop an accurate closed-form approximate solution to the model. Our results indicate that declining prices lead to substantial decrease in the optimal cycle time and much frequent ordering. This explains the heavy emphasis on just-in-time inventory management practiced by successful companies in high-tech industries. While previous models attributed the success of just-in-time policies to reduced holding cost and improved quality, under declining prices a substantial source of savings becomes lower costs of raw materials which is significant part of cost in these industries. We illustrate the results of the model with a numerical example and perform sensitivity analysis.

Keywords: Inventory policy; Continuous price change; High-tech industries

1. Introduction and literature review

The economic order quantity (EOQ) model of Harris [1] has been frequently extended by relaxing some of the original assumptions used. One of the key assumptions is that all costs in the model do not change during the foreseeable horizon. This assumption does not reflect the situation where inflation rate is high or the situation where price increase or decrease is expected. Several extensions have been developed for these cases, in which any or all of the cost parameters (ordering cost, unit purchase cost and holding cost) are not fixed. In today's high-tech industries, in particular, the personal computer (PC) assembly industry, we observe that components' cost is decreasing at a sustained and significant rate. However, existing literature does not provide a solution to the problem of determining the optimal order quantity under continuous sustained price decrease. Hence, in this paper we develop an EOQ model that considers continuous purchase price decrease and price-dependent holding cost. We derive the optimality condition along with a closed-form approximate solution for the optimal cycle time, test the accuracy of the approximation, and demonstrate that it provides satisfactory solutions for any reasonable problem parameters.

The problem is motivated by the frequent discussion in business media of the success of Dell Computer Corporation, which achieved unprecedented growth through efficient inventory management [2,3]. In 2001, Dell lowered their days of supply in inventory to 5 days [4] and this supply is still decreasing in 2002. Compaq, on the other hand, kept more than 20 days of inventory [5] and finally have been merged into Hewlett–Packard. In the PC industry, some components' cost is declining about 1% per week [2]. This implies that purchasing components one week earlier than the competition will result in an extra 1% increase in materials cost, aside from the increase in the holding cost.

An in-depth look at the success story of Dell reveals two strategies that they used to achieve double-digit growth rate for several years: build-to-order and just-in-time (JIT)

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doi:10.1016/j.omega.2003.08.009

inventory. Apple computer Inc., which achieved a remarkable 2 days of supply in inventory, also follows the same model [6,7]. Gateway Inc., although not successful in making profits in recent years, also mimics Dell's build-to-order and direct sales models, and carries about 8 days of supply in inventory [8]. A large part of these achievements in carrying low inventory can be attributed to the build-to-order strategy, in which no product is manufactured before a customer order is in place. JIT practice is a critical part of this build-to-order strategy. Hence, it is very likely that many suppliers will gather in close proximity to a manufacturer's plant and deliver raw materials supplies as they are needed, and such is the case with Dell. However, the essential question of how much to order (or alternatively how often to order) still remains.

Existing EOQ model extensions developed enhance the usability of the model by relaxing assumptions or by including new constraints. EOQ models which allow price change can be summarized based on two criteria: finite horizon vs. infinite horizon and continuous price change vs. single announced price change (see Table 1). Infinite horizon EOQ models with a single price change appear to have been first studied by Naddor [9]. Naddor's model and other similar infinite horizon models in textbooks (e.g., Peterson and Silver [10]) assume that price change occurs at the end of an EOQ cycle. These models have been extended by Taylor and Bradley [11] by considering a situation where the price increase does not coincide with the end of EOQ cycle. Goyal et al. [12] presented a review of models that considers announced future price increase.

In order to consider the lifecycle of a product, finite horizon models have also been studied in several papers. Optimal inventory policies for finite horizon with price changes are obtained by Lev and Soyster [13], Goyal [14] and Markowski [15]. Later, Lev and Weiss [16], whose algorithm to find optimal policy has been improved by Goyal [17] and Gascon [18], presented the structure of optimal inventory policy for both finite and infinite horizon models with single change in any or all costs in the classic EOQ model. Lev and Weiss [16] showed that the solutions for infinite horizon cases can be derived as limiting case of the finite horizon.

In contrast to single price change models, a few continuous price change models exist in the inventory management literature. Buzacott [19] and Erel [20] considered continuous price increase due to inflation. The two models are similar except for the treatment of the setup cost and the planning horizon. Buzacott's model assumed compound increasing price and setup cost in an infinite horizon, whereas Erel's model considered a compound increasing price during a finite planning horizon.

Although there have been many EOQ extensions with price changes, only a few considered the possibility of price decrease, and they are mostly limited to the single price change case (Goyal et al. [12], Lev and Weiss [16]). To the best of our knowledge, there have been no attempts to consider continuous price decrease in the EOQ literature. While Buzacott's [19] and Erel's [20] models can be used for price decrease, they require additional assumptions which make them unsuitable for that purpose. Buzacott's model works only if the same rate of price change occurs to the ordering cost. Therefore, this model's application to the price decrease case requires that the ordering cost be decreasing at the same rate as product unit price, which is not realistic. Furthermore, by assuming an increasing ordering cost, which offsets the increasing price effects, the fixed cycle time used by Buzacott becomes optimal or near optimal. On the other hand, increasing or decreasing product prices accompanied by a fixed ordering cost may make an increasing or decreasing cycle time, at least periodically, optimal. Erel's model which is intended to deal with high inflation can also be used for the price decrease case, however, his approximation is not accurate for inflation rates of less than 10% for the problem parameters in his example and the approximation was shown to have significant errors [21]. In addition, he does not test the quality of the approximate solution for different problem parameter combinations. Also, his formula for the approximate optimal order quantity requires many calculations and is much more complex than our approximation which is demonstrated to be very accurate for many combinations of possible problem parameters (the numerical experiment showed an average penalty of 0.000031% above the global minimum total cost due to the approximation and the worst case penalty of 0.0011%). Therefore, the

Table 1
EOQ models with price change

EOQ models with price changes	Finite horizon	Infinite horizon	Continuous price change (increase)	Single price change
Naddor [9], Peterson and Silver [10], Taylor and Bradley [11], Goyal et al. [12]		√		√
Lev and Soyster [13], Goyal [14], Markowski [15]	√			√
Lev and Weiss [16], Goyal [17], Gascon [18]	√	√		√
Buzacott [19]		√	√	
Erel [20]	√		√	