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The stochastic dynamic production/inventory lot-sizing problem with service-level constraints

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Abstract

This paper addresses the multi-period single-item inventory lot-sizing problem with stochastic demands under the "static–dynamic uncertainty" strategy of Bookbinder and Tan (Manage. Sci. 34 (1988) 1096). In the static-dynamic uncertainty strategy, the replenishment periods are fixed at the beginning of the planning horizon, but the actual orders are determined only at those replenishment periods and will depend upon the demand that is realised. Their solution heuristic was a two-stage process of firstly fixing the replenishment periods and then secondly determining what adjustments should be made to the planned orders as demand was realised. We present a mixed integer programming formulation that determines both in a single step giving the optimal solution for the "static–dynamic uncertainty" strategy. The total expected inventory holding, ordering and direct item costs during the planning horizon are minimised under the constraint that the probability that the closing inventory in each time period will not be negative is set to at least a certain value. This formulation includes the effect of a unit variable purchase/production cost, which was excluded by the two-stage Bookbinder–Tan heuristic. An evaluation of the accuracy of the heuristic against the optimal solution for the case of a zero unit purchase/production cost is made for a wide variety of demand patterns, coefficients of demand variability and relative holding cost to ordering cost ratios. The practical constraint of non-negative orders and the existence of the unit variable cost mean that the replenishment cycles cannot be treated independently and so the problem cannot be solved as a stochastic form of the Wagner–Whitin problem, applying the shortest route algorithm.

Keywords: Inventory; Chance-constrained programming; Integer programming; Modelling; Optimisation

1. Introduction

The study of lot-sizing began with Wagner and Whitin (1958), and there is now a sizeable literature in this area extending the basic model to consider capacity constraints, multiple items, multiple stages, etc. However, most previous work on lot-sizing has been directed towards the

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deterministic case. The reader is directed to De Bodt et al. (1984), Potts and Van Wassenhove (1992), Kuik et al. (1994) and Kimms (1997) for a review of lot-sizing techniques.

The practical problem is that in general much, if not all, of the future demands have to be forecast. Point forecasts are typically treated as deterministic demands. However, the existence of forecast errors radically affects the behaviour of the lot-sizing procedures based on assuming the deterministic demand situation. Forecasting errors lead both to stockouts occurring with unsatisfied demands and to larger inventories being carried than planned. The introduction of safety stocks in turn generates even larger inventories and also more orders. It is reported by Davis (1993) that a study at Hewlett-Packard revealed the fact that 60% of the inventory investment in their manufacturing and distribution system is due to demand uncertainty.

There has been increasing recognition as illustrated by Wemmerlov (1989) that future lot-sizing studies need to be undertaken on stochastic and dynamic environments that have at least a modicum of resemblance to reality. Inevitably, the forecast errors have to be taken into account in planning the future lot-sizes. Similar concerns have been expressed by Silver: "One should not necessarily use a deterministic lot-sizing rule when significant uncertainty exists. A more appropriate strategy might be some form of probabilistic modelling."

Silver (1978) suggested a heuristic procedure for the stochastic lot-sizing problem assuming that the forecast errors are normally distributed. A similar heuristic, having a different objective function, was presented by Askin (1981). Bookbinder and Tan (1988) proposed another heuristic, under the "static–dynamic uncertainty" strategy. In this strategy, the replenishment periods are fixed at the beginning of the planning horizon and the actual orders at future replenishment periods are determined only at those replenishment periods depending upon the realised demand. The total expected cost is minimised under the minimal service-level constraint. In this paper, we propose a mixed integer programming formulation to solve the stochastic dynamic lot-sizing problem to optimality under the "static–dynamic uncertainty" strategy of Bookbinder and Tan.

The optimal solution to the problem is the (s, S) policy with different values for each period in the time varying demand situation. Where the demand level changes slowly, it is usually satisfactory to use a steady-state analysis with constant S and s values updated once per year. However, this is inappropriate if the average demand can change significantly from period to period. This presents a non-stationary problem where the two control parameters change from period to period. The uncertainty in the timing of future replenishments caused by an (s, S) policy may be unattractive from an operational standpoint.

Although the inventory problems with stationary demand assumption are well known and extensively studied, very little has appeared on the non-stationary stochastic demand case. Recently, Sox (1997) and Martel et al. (1995) have described static control policies under the non-stationary stochastic demand assumption in a rolling horizon framework. Sox (1997) presents a mixed integer non-linear formulation of the dynamic lot-sizing problem with dynamic costs, and develops a solution algorithm that resembles the Wagner–Whitin algorithm but with some additional feasibility constraints. Martel et al. (1995) transform the multiple item procurement problem into a multi-period static decision problem under risk. Other notable works on non-stationary stochastic demand adopt (s, S) or base-stock policies and are due to Iida (1999), Sobel and Zhang (2001), and Gavirneni and Tayur (2001). In Iida (1999), the periodic review dynamic inventory problem is considered and it is shown that near myopic policies are sufficiently close to optimal decisions for the infinite horizon inventory problem. In Sobel and Zhang (2001), it is assumed that demand arrives simultaneously from a deterministic source and a random source, and proven that a modified (s, S) policy is optimal under general conditions. Gavirneni and Tayur (2001) use the derivative of the cost function that can handle a much wider variety of fluctuations in the problem parameters. The above studies have adopted either static control policies in a rolling horizon framework or dynamic control policies