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Multi-product capacity-constrained lot sizing with economic objectives

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Abstract

The capacity-constrained lot-sizing problem with stochastic lot arrivals is especially relevant within batch production environments. Progress has previously been made in minimizing queue times and flow times under generalized queueing assumptions. This paper introduces extensions incorporating economic criteria for the single machine case. Holding costs related to queueing delays are considered along with traditional EOQ cost components. Relationships are developed to determine lot sizes that minimize costs for the single and multiple product cases when the production rate is specified. As well, relationships to determine both lot sizes and throughput rates that maximize profits are examined.

Keywords: Stochastic lot sizing; Capacitated lot sizing; Multi-product lot sizing; Cost minimization; Profit maximization

1. Introduction

Batch manufacturing systems usually have a significant amount of work-in-process inventory. This inventory often resides as lots within queues ahead of capacity constrained resources. Although there has been considerable interest in lot sizing research, most procedures developed are not applicable to capacity-constrained problems with stochastic lot arrivals. In this research we are particularly interested in the problem of a single machine processing multiple product types in lots under generalized interarrival time assumptions. The objective is to optimize lot sizes based on economic criteria. Cost structures similar to those used in the traditional Economic Order Quantity (EOQ) formulation are considered. However, the holding costs associated with work-in-process inventory resulting from stochastic arrivals and capacity-constrained resources are also considered.

Several lot sizing studies have considered stochastic lot interarrival times with cost minimization as an objective. Williams (1984) noted that classical inventory control generally ignores capacity constraints and interactions between products. He considered an M/G/m queuing model that incorporated inventory holding costs, setup costs and backorder costs. Lot sizes and reorder points were treated as the decision variables in cost minimization. As well, Williams addressed the problem of using demand patterns to decide

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whether to make products to order or to stock. Bertrand (1985) presented a cost minimization model that considered setup costs and both work-in-process (WIP) and finished goods inventory holding costs. Optimal lot sizes were determined through iteratively estimating lot flow times and then readjusting lot sizes until convergence occurred. Flow time estimation was based on closed queueing network assumptions and the lot size optimization model was based on solving a set of non-linear equations, derived through partial differentiation of the cost function with respect to product lot sizes. Jönsson and Silver (1985) considered the same types of costs in analyzing the single-product, single-machine problem where Poisson demand for items results in lot arrivals that fall into an Erlang distribution. The multi-product case was also examined under M/D/1 queuing assumptions. Zipkin (1986) presented an approach that included setup costs, WIP and finished goods holding costs, and backorder costs. Estimates for both the mean and variance of lot flow times are required. Models to minimize costs under M/M/1, M/G/1 and Jackson queueing network assumptions were illustrated. Karmarkar (1987) also included a cost formulation for the M/M/1 problem but only considered WIP and finished goods holding costs. Finally, Missbauer (2002) discusses the need for capacity-constrained lot-sizing research including economic factors. He demonstrated the use of M/G/1 lot-sizing relationships which minimize the weighted sum of queue times, setup costs and finished goods holding costs.

The analytical approaches used in studies with economic objectives have frequently assumed a Poisson arrival process. This has the advantage of improving tractability with respect to determining lot queue time, which has a direct relationship to the amount of inventory in queue. However, in reality lot interarrival time distributions are generally not negative exponential. Although a model based on generalized queueing assumptions is of the most practical interest, there is no exact solution for expected queue times under GI/G/1 assumptions. Therefore, approximations are required to model inventory in queue. Most approximations are based on the mean and variance of the interarrival and service time distributions. Examples can be found in Kuehn (1979), Shanthikumar and Buzacott (1980), Whitt (1983) and Buzacott and Shanthikumar (1993). Although these approximations are generally presented in the context of single items being the entities in queue, they also apply when the entities are product lots.

Finding near optimal, or *approximated optimal*, lot sizes under GI/G/1 assumptions is important if queuing relationships are to be applied in real production analysis. Several recent studies have considered generalized lot interarrival times with objectives to minimize lot queue times or flow times. Lambrecht and Vandaele (1996) dealt with generalized lot interarrival times for a single product type at a single machine. Lot sizing considerations in their research included the time to accumulate lots as well as lot flow times at the machine. The solution procedure was based on a steepest decent search algorithm. A further contribution was the development of lot flow time distributions. Lambrecht et al. (1998) extended the development of this lot-sizing approach, as part of the ACLIPS scheduling procedure, to multiple product types moving through multiple machines. Enns and Choi (2002) used partial differentiation to determine optimal lot sizes, based on approximate flow time relationships. The lot-sizing relationships were then embedded in a materials requirements planning (MRP) system. Exploratory results demonstrated the lot sizing approach could be used effectively with time-phased planning.

In this research, the relationships derived in Enns and Choi (2002) are extended to deal with both cost minimization and profit maximization for the single machine problem. The next section presents the basic assumptions and problem formulation, while Section 3 examines lot flow time estimation. Sections 4 and 5 deal with cost minimization and profit maximization in the single product case. Sections 6 and 7 then extend these models to the multiple product case. Finally, results are discussed and conclusions drawn.

2. Problem formulation for the single-machine case

The problem being considered is illustrated in Fig. 1. This figure shows two types of products being processed at a single machine in lot sizes of Q_1 and Q_2. The lots entering the machine queue have an