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A finite horizon lot-sizing problem with time-varying deterministic demand and waiting-time-dependent partial backlogging

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Abstract

This paper presents a general time-varying demand inventory lot-sizing model with waiting-time-dependent backlogging and a lot-size-dependent replenishment cost. It differs from many related trended inventory replenishment models in two directions. First, our model not only allows part of the backlogged demands to turn into lost sales, but this backlog-to-lost-sales conversion rate is modeled by a general continuously decreasing function of the remaining waiting time until the next replenishment delivery. Second, this paper considers the dependence of replenishment cost on lot size. We derive the model's cost function for a "shortages followed by inventory" replenishment policy. Some convenient mathematical properties of the cost function are identified, with which an effective numerical solution procedure is developed for determining the optimal replenishment policy. Numerical examples and some sensitive-analysis results are then reported.

Keywords: Time-varying demand; Lot size; Time-dependent backlogging rate; Shortages

1. Introduction

Since Donaldson's (1977) pioneering work on a deterministic linear-demand-trend inventory model, many researchers have considered various aspects of an inventory system with time-varying demand patterns. For example, in order to reduce the computational complexity of Donaldson's model, Phelps (1980), Ritchie (1984), Goyal (1987), Yang and Rand (1993), Teng (1994), Hill (1995) (among others) proposed alternative solution techniques or heuristic approaches. Another group of researchers extended the preceding works by considering the occurrence of shortages (and hence shortage costs). Generally speaking, those models that deal with shortages can be divided into two categories. The first category (e.g., Murdeshwar, 1988; Dave, 1989a, b; Datta and

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Pal, 1992) uses an "inventory followed by shortages" (hereafter "IFS") replenishment policy; i.e., each replenishment cycle starts with a replenishment arrival (and hence a non-negative inventory level) and ends with shortages (although shortages are not permitted at the end of the last cycle of a finite planning horizon). The second category (e.g., Goyal et al., 1992; Hariga and Goyal, 1995) uses a "shortages followed by inventory" (hereafter "SFI") replenishment policy; i.e., each replenishment cycle (including the first cycle that begins at time 0) starts with a "stockout interval;" after sometime a replenishment delivery is received, thus initiating an "inventory interval." Recently, through an extensive empirical investigation, Goyal et al. (1996) showed that an SFI policy can often outperform an IFS policy in reducing the total system cost; however, various subsequently published lot-sizing procedures continue to consider only IFS policies.

All the preceding models assumed that the shortages could be completely backlogged. However, when shortages arise, very often some customers would cancel their orders (resulting in "lost sales") rather than wait for the next replenishment. Thus, Wee (1995) and Giri et al. (2000) (among others) modeled this lost sales as a fixed fraction of the shortages incurred. Chang and Dye (1999) and Wang (2002) further recognized that, when shortages arise, customers facing a long waiting time until the next replenishment are more likely to cancel their orders; therefore, they modeled the backlogging rate as a specific reverse-proportional function of the "waiting time" (i.e., time until the next replenishment).

All the models mentioned above assumed that the "fixed" replenishment cost per order remains constant for all the replenishment cycles. However, in many situations the replenishment cost incurred by retailers contains two components. One component consists of such items as administrative, communication and kindred costs; the other component is made up of such items as transportation, loading and unloading costs. While the first replenishment cost component is largely independent of the replenishment lot size, the second replenishment cost component frequently varies with the lot size. Nevertheless, under a constant demand rate the second component remains constant over different cycles because the optimal replenishment lot size should be the same for all order cycles. Under time-varying demands, different order cycles will have different optimal replenishment lot sizes, hence the second component will have different magnitudes in different cycles. Still, this second component can be ignored because the sum of these second cost components over the entire planning horizon would remain constant, as long as all demands are met either immediately or after backlogging. However, if partial lost sales occur, the sum of demands actually met—and hence the sum of the second cost components—will vary with the replenishment policy. Therefore, now the variation of replenishment costs over different cycles needs to be properly accounted for. Very recently, Bhunia and Maiti (1999) developed a deterministic linearly increasing-demand inventory model that considers such a lot-size-dependent replenishment cost structure. Zhou and Lau (2000) extended it to the case of a general time-varying demand, while Kar et al. (2001) extended it for a situation with two levels of storages. However, none of these three studies was applying their lot-size-dependent replenishment cost structures to situations with partial lost sales.

The purpose of this paper is to develop, for the situation in which the demand rate of the item follows a time-varying deterministic function, a more general lot-sizing procedure that incorporates a variety of new or recently published modeling features. Among the new features are the following. First, we use a partial backlogging model that is more general than the ones used in Chang and Dye (1999) and Wang (2002) mentioned above. Second, we model the dependence of the "fixed" replenishment cost on lot size—a hitherto overlooked factor which becomes relevant when the demand rate is not constant but varies over time. Among the recently published features, our model will be optimized with an SFI replenishment policy because of its known superiority over the alternative IFS policy. We charge backlogs under a cost/unit/year basis and charge lost sales separately under a cost/unit basis.

The rest of this paper is organized as follows. Notations and definitions are stated in the next