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Warehouse space capacity and delivery time window considerations in dynamic lot-sizing for a simple supply chain ☆

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Abstract

This paper studies a single item, two-echelon dynamic lot-sizing model with delivery time windows, early shipment penalties, and warehouse space capacity constraints. The two-echelon system consists of a warehouse and a distribution center. The underlying problem is motivated by third party logistics and vendor managed inventory applications in the computer industry where delivery time windows are typically specified by the distribution center under the terms of a supply contract. The capacity of the warehouse is limited. This constraint should be considered explicitly because the finished products are expensive items (such as computer equipment and peripherals), and they have to be stored in the warehouse in an appropriate climate before they are shipped to the distribution center. Studying the optimality properties of the problem, the paper provides a polynomial time algorithm for computing its solution. The optimal solution includes: (i) *the replenishment plan* specifying "when, and in what quantities, to replenish the stock at the third-party warehouse," and (ii) *the dispatch plan* specifying "when, and in what quantities, to release an outbound shipment to the distribution center, and in which order to satisfy the demands." The algorithm is based on dynamic programming and requires $O(T^3)$ computational complexity.

Keywords: Multi-echelon inventory; Dynamic lot-sizing; Demand time window

1. Introduction

Supply contracts with time window considerations are important for effective third party logistics (TPL) and vendor managed inventory (VMI) practices. For example, under a typical contract of this type in the computer industry in Texas, a TPL provider is in charge of the

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outbound distribution and VMI programs of the manufacturer. Hence, the inventory and demand information of the downstream distribution center (or DC) is accessible to the TPL provider. After reviewing the downstream inventory levels, the TPL provider is empowered to make decisions regarding the quantity/timing of re-supply (Çetinkaya and Lee, 2000). The distribution center, however, requests timely deliveries by imposing delivery time windows for shipments. A time window is basically a *grace period* during which a demand can be satisfied with no early or late shipment penalty. Naturally, such a system is favorable for effective VMI where the supplier is responsible for managing inventories at the downstream supply chain member and guaranteeing timely delivery by satisfying the time window constraints. This paper considers a generalized dynamic lot-sizing problem with delivery time window and warehouse capacity considerations arising in the applications described above.

In order to set the stage for a mathematical representation, let us consider a particular application of the problem with a single item. Suppose that finished products are shipped from the manufacturer to a third-party warehouse (TPW) for temporary storage and distribution. A linear inventory carrying cost is incurred for each unit held in inventory at the TPW per unit of time, and a fixed setup cost is incurred each time the stock is replenished at the TPW. The capacity of the TPW is limited. This constraint should be considered explicitly because the finished products are expensive items (such as computer equipment and peripherals), and they have to be stored in the warehouse in an appropriate climate. Products are delivered from the TPW to a distribution center (DC) in bulk replenishment quantities. For each dispatch to the DC from the TPW, again, a fixed setup/delivery cost is incurred. Also, each demand at the DC has a time window, specified under the terms of a contract between the TPW and DC, during which the demand can be satisfied without penalty. The time window of a demand specifies the earliest and latest delivery times requested by the DC. If a demand is delivered prior to its earliest delivery time, a linear pre-shipping penalty (holding cost at the customer level) is incurred for each unit per unit of time until the earliest delivery time is reached. Backlogging is prohibited since the DC already allows a time window, i.e., a grace period for delivery timing flexibility. That is, a demand cannot be delivered later than its latest delivery time. Finally, it is worth noting that we consider the case where both inbound replenishments and outbound dispatches are delivered instantaneously. However, the proposed solution approach is directly applicable to accommodate positive deterministic lead-times.

In summary, the problem of interest in this paper is a single item, two-echelon dynamic lot-sizing model with time window considerations and warehouse capacity constraints where the objective is to find an *integrated replenishment policy for the TPW and the DC simultaneously* to satisfy all demands at the DC at minimum cost. As for the traditional dynamic lot-sizing literature, the paper assumes that demand is known in advance. This class of models has a wide domain of applications where orders have been placed in advance or contracts have been signed ahead of time specifying deliveries for the next few periods.

In a recent paper, Lee et al. (2001) generalize the classical (single echelon) dynamic lot-sizing model to consider demand time windows, and they provide polynomial time algorithms for two cases—where backorders are allowed and where they are not. Here, we extend this recent paper by considering a two-echelon problem arising in a TPW where the warehouse capacity constraint can be modeled explicitly. The special case where warehouse capacity is ignored has been analyzed by Jaruphongsa et al. (2002). By modeling the warehouse capacity limit explicitly, this paper also extends the literature on dynamic lot-sizing models with inventory constraints (e.g., see Love, 1973; Erenguc and Aksoy, 1990; Sandbothe and Thompson, 1993; Toczylowski, 1995). Studying the optimality properties of the problem, the paper provides a polynomial time algorithm for computing the optimal solution. The algorithm is based on dynamic programming and requires $O(T^3)$ computational complexity.