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Optimal lot sizing for an unreliable production system under partial backlogging and at most two failures in a production cycle

B.C. Giri, W.Y. Yun*

Department of Industrial Engineering, Pusan National University, Pusan 609-735, South Korea

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Abstract

The paper considers an economic manufacturing quantity problem for an unreliable manufacturing system where the machine is subject to random failure and at most two failures can occur in a production cycle. Upon first failure, the repair action is started immediately and the demand is met first from the on-hand inventory. The shortages, if occur due to longer repair time, are backlogged partially by resuming the production run after machine repair. While backlogging, if failure occurs again then the accumulated shortages until completion of the second repair are assumed to be lost. The model is formulated under general failure and general repair time distributions. Some characteristics of the model with exponential failure and exponential/constant repair times are studied. The optimal lot sizing policies are derived and the sensitivity of some model parameters is examined with the help of numerical examples.

Keywords: Production; Lot sizing; Machine breakdowns; Partial backlogging

1. Introduction

The classical economic manufacturing quantity (EMQ) model determines the optimal lot size of a product by minimizing the total relevant costs per unit time assuming that the demand and production rates are deterministic, inventory holding and setup costs are known constants and the facility is failure-free. Numerous research efforts have been devoted to extend this basic EMQ model for close fitting to various real-world situations (see Naddor, 1984; Nahmias, 1989; Silver et al., 1998, for references). However, until late 1980s, much attention has not been paid by the researchers to the effects of uncertainties such as quality variation and machine reliability on the optimal lot sizing decision. Bielecki and Kumar (1988) made an attempt to study a production–inventory model where the inter-failure times and repair times are assumed to be exponentially distributed, the machine production rate is bounded by a fixed maximum limit and shortages, if occur after breakdown, are backlogged. They showed that there exist ranges of parameter values of the model for which zero-inventory policy can be optimal even when there is uncertainty in the

*Corresponding author. Tel.: +82-51-510-2421; fax: + 82-51-512-7603.
E-mail address: wonyun@hyowon.pusan.ac.kr (W.Y. Yun).

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manufacturing system. Posner and Berg (1989) developed an EMQ model with exponentially distributed failure and repair times and compound Poisson demand process. They computed several important system performance measures such as expected inventory stocked, losses due to shortages, machine's utilization, etc. Hopp et al. (1989) considered a production system consisting of two machines with an immediate buffer where only the first machine is subject to random failure. They used a (b, B) control policy in which the first machine starts production when the inventory level falls below b, and stops when inventory level reaches B. Assuming exponential failure and repair times, they obtained the values of b and B that minimize the relevant inventory costs. Groenevelt et al. (1992a) studied the effects of stochastic machine breakdown and repair on lot sizing decisions under two different situations. First, they assumed that after a machine breakdown, the machine setup is totally lost and the interrupted lot is aborted. The next production after machine repair starts when the on-hand inventory is depleted. Second, they considered the continuation of the production process after machine repair if the cost of resuming the production run is substantially lower than the setup cost. They showed that for exponential inter-failure time and negligible repair time, the optimal lot size in each model is independent of repair cost and increased with failure rate. In the subsequent article, they (Groenevelt et al., 1992b) extended their previous study by assuming that the inter-failure times are exponentially distributed and that the repair times are i.i.d. random variables with a general probability distribution. Moreover, they assumed that a certain fraction of the items produced is diverted into a safety stock while the remaining fraction is used to meet the current demand. This safety stock is depleted whenever the machine undergoes repair after a breakdown. They developed bounds on the range of feasible service levels and investigated the impact of several system parameters on this range. Kim and Hong (1997) and Kim et al. (1997) generalized Groenevelt et al. (1992a)'s model assuming arbitrarily distributed inter-failure times. Chung (1997) determined the bounds for the optimal lot sizes in Groenevelt et al. (1992a)'s model. Later, he (Chung, 2003) provided better bounds for the EMQ and also showed that the long-run average cost function in Groenevelt et al. (1992a)'s model is unimodal, though it is neither convex nor concave. Berg et al. (1994) analyzed a production system with multiple identical machines devoted to produce a single part type by employing level crossing techniques. They computed performance measures, similar to Posner and Berg (1989), that characterize the operation of the production–inventory system. Abboud (1997) presented a simple approximation of the EMQ model with Poisson machine breakdowns and general repair times where the demand is satisfied from on-hand inventory during repair time and if stock out situation occurs due to longer repair time, demands are met from other source(s) at a higher cost. For an unreliable manufacturing system, Dohi et al. (1997) derived the EMQ policy which can be characterized as an age replacement like policy. They formulated the expected cost function under general failure and repair mechanisms and derived some properties on the optimal policy under certain failure and repair circumstances. Makis and Fung (1998) investigated the joint effect of process deterioration and random machine failure. They developed an EMQ model with generally distributed machine failure and repair times and periodic inspections for the production process that may shift to an out-of-control state at any random time. Moini and Murthy (2000) developed an EMQ model for an unreliable production system under Type I and Type II repair action strategies, assuming that the probabilities of machine failure after Type I and Type II repairs are not the same. Moreover, the costs for the two types of repair are different. By analyzing the model they tried to find a relationship between process uncertainty, repair actions and lot size. Abboud (2001) modeled a production–inventory system as a Markov chain assuming time to be discrete units and the machine failure and repair times to be geometrically distributed. He developed an efficient algorithm to compute the potentials of the Markov chain which were used to construct the cost model. Dohi et al. (2001) investigated the joint implementation of preventive maintenance and safety stocks in an unreliable production environment. Recently, Giri and Dohi (2004) implemented the net present value (NPV) approach to compute the EMQ for a failure-prone production facility. They compared the performance of the NPV model and the traditional long-run