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A note on "Khouja and Park, optimal lot sizing under continuous price decrease, Omega 31 (2003)"

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Abstract

Khouja and Park [1] analyze the problem of optimizing the lot size under continuous price decrease. They show that the classic EOQ formula can lead to far from optimal solutions and develop an alternative lot size formula using the software package Mathematica. This formula is more exact, but also more complicated. In this note, we study the net present value formulation of the model, and thereby gain an insight that leads to the proposal of a modified EOQ formula. The modified EOQ formula, albeit not as accurate, is a good alternative to the formula developed by Khouja and Park, especially if mathematical complexity may hamper implementation.

Keywords: Lot sizing; Economic order quantity; Price decrease

1. Introduction

Khouja and Park [1] analyze the problem of optimizing the lot size under continuous price decrease. This problem is relevant for the high-tech industry and especially the PC assembly industry, where the prices of components decrease at significant rates. They study a single-item model with a constant lead time, constant demand, no quantity discounts, and no shortages allowed. However, their model deviates from the standard economic order quantity (EOQ) model in two ways: (i) there is a finite planning horizon, and (ii) the purchase price decreases at a constant rate.

Khouja and Park develop an expression for the total cost over the planning horizon using a mixture of the average cost (AC) approach and the net present value (NPV) approach. They continuously discount the price as in an NPV approach, but charge an interest cost per time unit rather than discount purchase cost. By setting the derivative of the cost expression to zero and using a Taylor series approximation for one of the exponential terms, they derive a complex optimality condition for the number of orders during the planning horizon. Using the software package Mathematica, they then find an expression for the number of orders during the planning horizon, which leads to nearly optimal solutions for realistic values of the model parameters. They also develop the corresponding expression for a nearly optimal order quantity.

For a specific example, Khouja and Park illustrate that their order quantity formula indeed leads to a nearly optimal solution. They further show for this example, that the classic EOQ formula, with holding cost per unit of inventory value per time unit equal to the interest rate, results in a far from optimal solution.

As mentioned above, Khouja and Park use a mixture of the AC approach and the NPV approach in deriving their total cost expression. In this note, we instead develop a 'pure' NPV expression. Although the numerical difference between the expressions is small for examples with realistic parameter settings, the pure NPV expression leads to the

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important insight that the holding cost per unit of inventory value per time unit in the ‘corresponding’ AC approximation is equal to the interest rate plus the rate of price decrease. We therefore propose a modified version of the classic EOQ formula with holding cost per unit of inventory value per time unit equal to the interest rate plus the rate of price decrease.

We illustrate for the example of Khouja and Park, that the modified EOQ formula leads to a nearly optimal solution. An extensive numerical experiment shows that this result also holds in general. Combining this near-optimality with the simple structure of the EOQ formula that many practitioners are familiar with, we conclude that the modified EOQ formula has great practical value.

The remainder of this paper is organized as follows. In Sections 2 and 3, we review the model and the results of Khouja and Park [1]. In Section 4 we apply the pure NPV approach and present our results. We end with conclusions in Section 5.

2. The model of Khouja and Park

The model of Khouja and Park is based on the following standard EOQ assumptions.

- No quantity discount are given.
- The lead time is constant.
- No shortages are allowed.
- The demand rate is constant.

But the model differs from the classical EOQ model in the following ways.

- The price decreases at a constant percentage over time.
- The planning horizon is finite.

The objective is to minimize the total cost over the planning horizon. Relevant costs are: the ordering cost (per order), the purchase cost (per unit of product), and the holding cost (per unit of inventory value per time unit). Note that the entire holding cost is expressed per unit of inventory value. From now on, we will refer to this cost as the interest cost. This will avoid confusion in Section 4, where we argue that the holding cost should also include the rate of price decrease.

The notations that Khouja and Park use are listed in .Table 1 That table also includes additional notations that will be used in Section 4.

Table 1
Notation

D	Demand per unit of time
S	Ordering cost
r	Interest cost per unit of inventory value per unit of time
a	Continuous interest rate corresponding to r
u	Per cent price decrease per unit of time
b	Continuous price decrease rate corresponding to u
T	Length of the planning horizon
n	Number of orders during the planning horizon
C_0	Price per unit of product at time 0

3. The results of Khouja and Park

Since the price of the product is C_0 at time 0 and decreases by u per cent per time unit, it holds that

$$C(t) = C_0 e^{-bt},$$

where

$$b = -\ln(1 - u/100) \tag{1}$$

is the *continuous price decrease rate*.

The total cost over the planning horizon is therefore

$$\mathrm{TC} = nS + \sum_{i=0}^{n-1} \left[\frac{DTC_0 e^{-biT/n}}{n} + e^{-biT/n} r C_0 \int_{s=0}^{T/n} (T/n - s) D \, \mathrm{d}s \right]. \tag{2}$$

Khouja and Park show that an approximate continuous minimizer of TC is

$$\tilde{n} = \sqrt{\frac{C_0 DT(b+r)(e^{bT} - 1)}{2e^{bT} bS}} - \frac{bT}{2} \tag{3}$$

with corresponding order quantity

$$\tilde{Q} = \frac{DT}{\tilde{n}} \frac{2e^{bT/2} D\sqrt{bST}}{\sqrt{2C_0 D(b+r)(e^{bT} - 1)} - be^{bT/2}\sqrt{bST}}. \tag{4}$$

However, since n should be discrete, Khouja and Park propose to round $\tilde{n}$ to the closest integer and adjust the order quantity accordingly. This will be illustrated in an example in the next section. In that example, the optimal order quantity (4) is also compared to the classic EOQ defined by Khouja and Park as

$$\mathrm{EOQ}_c = \sqrt{\frac{2SD}{rC_0}}. \tag{5}$$

We remark that to avoid dependency of the classic EOQ on the unit of time, an alternative definition is

$$\mathrm{EOQ}'_c = \sqrt{\frac{2SD}{aC_0}}, \tag{6}$$

where, similar to (1), the continuous interest rate a is defined as (note that r is not defined as a percentage)

$$a = -\ln(1 - r). \tag{7}$$