A New Algorithm for Optimally Determining Lot-Sizing Policies for a Deteriorating Item in an Integrated Production-Inventory System

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Abstract—In this study, we focus on optimally determining lot-sizing policies for a deteriorating item among all the partners in a supply chain system with a single-vendor and multiple-buyers so as to minimize the average total costs. We revise Yang and Wee's [1] model using the Fourier series to precisely estimate the vendor's inventory holding costs. Also, we transform our revised model into a more concise version by applying an approximation to the exponential terms in the objective function. In order to solve this problem, we analyze the optimality structure of our revised model and derive several interesting properties. By utilizing our theoretical results, we propose a search algorithm that can efficiently solve the optimal solution. Based on our numerical experiments, we show that the proposed algorithm outperforms the existing solution approach in the literature, especially when the number of buyers is larger in the supply chain system. © 2006 Elsevier Ltd. All rights reserved.

Keywords—Integrated system, Search algorithm, Lot-sizing policy, Deterioration.

1. INTRODUCTION

This study aims at optimally coordinating lot-sizing policies for a deteriorating item among all the partners in a supply chain system with a single-vendor and multiple-buyers so as to minimize the average total costs. The vendor (which is a producer) distributes a deteriorating item to the buyers. We assume that the replenishment cycle of each buyer, denoted by T_i, must be an integer-ratio fraction of the replenishment cycle of the vendor (denoted by T). That is, $T_i = T/n_i$ and $n_i \in \{1, 2, 3, 4, \dots\}$ for all i.

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Deterioration occurs for most products in the real world. (We note that deterioration means that a product fails to regularly implement its function.) Ghare and Schrader [2] classified the deteriorating properties of inventory into three categories:

(1) direct spoilage, e.g., vegetable, fruit, and fresh food, etc.;
(2) physical depletion, e.g., gasoline and alcohol, etc.;
(3) deterioration such as radiation changing, negative spoiling, and loss of efficacy in inventory, e.g., electronic components and medicine.

From another point of view, deterioration can also be classified by the time-value or the products' life of inventory. Raafat [3] categorized deterioration by the time-value of inventory.

(1) Utility Constant: Its utility does not change significantly as time passes within its valid usage period, e.g., liquid medicine.
(2) Utility Increasing: Its utility increases as time passes, e.g., some alcoholic drinks.
(3) Utility Decreasing: Its utility decreases as time passes, e.g., vegetables, fruits, and fresh foods, etc.

On the other hand, Nahmias [4] classified deterioration by products' lifetime of inventory.

(1) Fixed Lifetime: Products' lifetime is prespecified and its lifetime is independent of the deteriorated factors; therefore, it is called *time-independent* deterioration. In fact, the utility of these products decreases during its lifetime, and when passing its lifetime, the product will perish completely and become of no value, e.g., milk, inventory in blood bank, and food, etc.
(2) Random Lifetime: There is no specified lifetime for these products. The lifetime for these products is assumed as a random variable, and its probability distribution could be a gamma distribution, Weibull distribution, or an exponential distribution, etc. Products that keep deteriorating in some probability distribution are also the so-called *time-dependent* deteriorating products, e.g., electronic components, chemicals, and medicine, etc.

The scope of this study covers those deteriorating products being classified as utility decreasing (as regards their time-value) and also as random lifetime (as regards their lifetime). Furthermore, we assume the deterioration of inventory to be exponentially distributed.

Since deterioration will incur additional costs for inventory storage, it could distort the decision-making scenario and mislead the decision makers' replenishment strategy if one ignores the deteriorating factor in their inventory models. However, most of the inventory models have considered the deteriorating factor as single-product or single-vendor single-buyer models, for instance, [5–9]. In the literature, the present authors have found very few articles that studied inventory models with multiple deteriorating products or single-vendor multibuyer models. Hwang and Moon [10] presented a production-inventory model that integrates the production planning of two products produced on a single facility and the raw material may be deteriorating over time with a constant rate. Kar *et al.* [11] proposed an inventory model for several continuously deteriorating products, sold from two shops under single management dealing with limitations on investment and total floor-space area. On the other hand, the one-warehouse multiretailer problem is one of the most representative studies in the integrated lot-sizing problems. One may refer to the following papers for further reference, namely, [12–16], etc. We note that these papers do not take into account the deteriorating factor in their mathematical models. Recently, some researchers have been working on the integrated lot-sizing models for a deteriorating item in single-vendor and multiple-buyers production-inventory systems. One may refer to [1,17–21] for reference. These inventory models share some common characteristics with the multiple-product inventory models though there still exist significant differences between them, especially in their solution approaches.

In this study, we focus on solving the inventory control problem presented in Yang and Wee's [1] paper. First, we review the assumptions in Yang and Wee's model as follows. There are totally N buyers in this supply chain system. Customer demand occurs with each buyer at a constant rate.