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# Single item optimal lot sizing under continuous unit cost decrease

Moutaz Khouja[a,*], Suresh Goyal[b]

[a]Business Information Systems and Operations Management Department, The Belk College of Business Administration, The University of North Carolina at Charlotte, Charlotte, NC 28223, USA

[b]Faculty of Commerce and Administration, Department of Decision Sciences and MIS, Concordia University, 1455, De Maisonneuve Blvd. West, Montreal, Quebec, Canada, H3G 1M8

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**Abstract**

An important characteristic of high-tech industries is decreasing component unit cost over time. In this paper, we develop an inventory model for products experiencing continuous decrease in unit cost. Unlike earlier models which were restricted to equal cycle times, we allow cycle times of varying length. We develop a fast algorithm for identifying an optimal or near-optimal solution to the model and test the performance of the algorithm. We illustrate the models with numerical examples.


*Keywords:* Inventory policy; Continuous unit cost change

## 1. Introduction and literature review

Recently, Khouja and Park (2003) developed a model for determining an economic operating policy for a product which has a declining unit cost over a finite time horizon. The unit cost of the product is continuously decreasing (or increasing) by a constant percentage over time. They developed an approximate closed-form expression for the optimal cycle time under the restriction of equal cycle times for the entire finite horizon and tested the accuracy of the approximation.

The model by Khouja and Park (2003) is one of the many models dealing with unit cost changes in inventory management literature. Since Khouja and Park reviewed many of those models in their paper, we only highlight some of the more relevant papers in our review. In contrast to single unit cost change models, a few continuous unit cost change models exist in the literature. Buzacott (1975) and Erel (1992) considered continuous unit cost

*Corresponding author. Tel.: +1 704 687 3242; fax: +1 704 687 6330.

*E-mail addresses:* mjkhouja@email.uncc.edu (M. Khouja), sgoyal@vax2.concordia.ca (S. Goyal).

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increase due to inflation. The two models are similar except for the treatment of the setup cost and the planning horizon. Buzacott's model assumed compound increasing unit cost and setup cost in an infinite horizon, whereas Erel's model considered a compound increasing unit cost during a finite planning horizon.

Few economic order quantity (EOQ) model extensions considered the possibility of a single decrease in product unit cost (Goyal, et al., 1991; Lev and Weiss 1990). While Buzacott's (1975) and Erel's (1992) models can be used for unit cost decrease, they require additional assumptions which make them unsuitable for that purpose. Buzacott's model works only if the same rate of unit cost change occurs to the ordering cost. Therefore, application of Buzacott's model to the unit cost decrease case requires that the ordering cost be decreasing at the same rate as unit cost, which may not be realistic. Furthermore, by assuming an increasing ordering cost, which offsets the increasing unit cost effects, the fixed cycle time used by Buzacott becomes optimal or near optimal. On the other hand, increasing or decreasing product unit cost accompanied by a fixed ordering cost may make varying cycle times optimal. Erel's model which is intended to deal with high inflation can also be used to deal with unit cost decrease. However, his approximation is not accurate for inflation rates of less than 10% for the problem parameters in his example and may have significant errors in general (Wilson, 1993).

More recently, Teng and Yang (2004) developed an EOQ model extension for products whose cost and demand are fluctuating over time. The model allows for partial backlogging and deals with finite horizon. For a given value of the number of replenishment cycles, the authors identified the conditions under which there is only one unique solution to the necessary optimality conditions for the durations of cycles. They also established an estimation procedure for the optimal number of replenishment cycles. Our model can be viewed as a special case of their model. We deal with constant demand but decreasing unit cost and no shortages allowed. Our algorithm therefore does not require integration and is simple to apply. In addition, we are able to treat the holding cost as a fraction of unit cost, which for cases where unit costs are declining rapidly, can have significant impact on optimal inventory policy.

In both models by Khouja and Park (2003) and Erel (1992) the cycle time were restricted to having equal length for the entire horizon. Khouja and Park assert that this is a good approximation when the planning horizon is relatively short and will deteriorate as the horizon gets longer. In addition, both models deal with unit costs which decrease at a constant percentage. In this paper, we remove the restriction of equal cycle times. We develop and solve models which allow varying cycle times. In the first model, we deal with linear decrease in unit cost and in the second model we deal with a constant percentage decrease in unit cost. In Section 2 of the paper, we deal with linear unit cost decrease. In Section 3, we deal with the constant percentage unit cost decrease. In Section 4, we present a numerical example and test the performance of the algorithm. We close in Section 5 with some concluding remarks and some possible research extensions.

## 2. Optimal inventory policy under linear decrease in unit cost

The assumptions of the classical EOQ model being used here to develop our model are:

- Demand is deterministic and uniform over the planning horizon.
- Quantity discounts are not available.
- The entire order quantity is delivered in one shipment.
- Shortages are not allowed.
- Lead time is constant.

**Notations**

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| $D$ | demand per unit time |
| $s$ | ordering cost |
| $r$ | fraction holding cost of inventory value per unit time |
| $T$ | length of the planning horizon |