

# Optimal production lot sizing with rework, scrap rate, and service level constraint

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Received 16 March 2006; received in revised form 20 November 2006; accepted 21 November 2006

## Abstract

This paper studies the optimal lot-sizing decision for a production system with rework, a random scrap rate, and a service level constraint. In most real-life manufacturing settings, generation of defective items is inevitable. Reworking of defective items may significantly reduce overall costs. We derive an optimal operating policy and prove that the expected overall costs of such a production system with backlogging permitted is less than or equal to that of the same model without backlogging. Then the relationship between the “imputed backorder cost” and maximal permitted shortage level is derived for decision-making on whether the required service level is achievable. In the case that the required service level is not attainable, an equation for calculating the intangible backorder cost is proposed. This enables us to derive a new optimal lot-size policy that minimizes expected overall costs as well as satisfies the service level constraint. A numerical example is provided to demonstrate its practical usage.

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*Keywords:* Production lot-size; Rework; Random defective rate; Service level constraint; Inventory model

## 1. Introduction

The economic order quantity (EOQ) model was first introduced several decades ago [1]. It is a mathematical method used to balance inventory setup and holding costs and derives an optimal order quantity that minimizes overall inventory expenses. Regardless of its simplicity, it is still applied industry-wide today [2,3]. In the manufacturing sector, the economic production quantity (EPQ) model is often used to determine the optimal lot-size for items that are produced internally instead of being purchased from an outside supplier. The classical EPQ model assumes implicitly that items produced are of perfect quality. However, in real-life situations, due to process deterioration or other factors, generation of defective items is inevitable. The imperfect quality items, sometimes, can be reworked and repaired; hence the overall production-inventory costs may be reduced significantly [4,5]. For examples, the printed circuit board assembly (PCBA) in PCBA manufacturing, plastic goods in the plastic injection molding process, and production process in other industries, such as metal components, textiles, etc., sometimes employ rework as an acceptable process in terms of level of quality.

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Recently, research has been undertaken to address the imperfect quality issue of production systems [4–8]. Examples are surveyed as follows. Rosenblatt and Lee [8] proposed an EPQ model that deals with imperfect quality. They assumed that at some random point in time the process might shift from an in-control to an out-of-control state, and a fixed percentage of defective items are produced. Approximate solutions for obtaining an optimal lot size were developed in their paper. Groenevelt et al. [9] proposed two production control policies to deal with machine breakdowns. The first policy assumes that production will not resume (NR policy) after a breakdown. In the second policy, production is immediately resumed after a breakdown if the on-hand inventory is below a certain threshold level (AR policy). Both policies assume the repair time is negligible and they studied the effects of machine breakdowns and corrective maintenance on economic lot sizing decisions. Zhang and Gerchak [10] considered joint lot sizing and inspection policy in an EOQ model with random yield. Cheng [11] formulated inventory as a geometric program and obtained closed-form optimal solutions for an EOQ model with demand-dependent unit production cost and imperfect production processes. Chung [12] studied bounds for production lot sizing with machine breakdown conditions. Hayek and Salameh [5] derived an optimal operating policy for the finite production model under the effect of reworking of imperfect quality items. They assumed that all of the defective items are reworked and repaired. Chiu [4] examined a finite production model with backlogging and rework. He assumed that a portion of defective items are scrap and the other portions are all repairable. An optimal lot size that minimizes overall inventory costs is derived accordingly.

In this paper, we consider the problem of a production system with both imperfect production and imperfect rework processes, scrap items, and service level constraints. In realistic production-inventory control, due to internal orders of parts/materials and other operating considerations, planned backlogging is a strategy to effectively minimize overall production-inventory costs. While allowing backlogging, abusive shortage in an inventory model, however, may cause an unacceptable service level and turn into possible losses of future sales (because of the loss of customer goodwill). Therefore, the minimal service level per cycle is always set as an operating constraint while deriving the optimal lot-size.

A considerable amount of research has been carried out to incorporate the service level constraint [13–17]. Examples are surveyed below. Bertsimas and Paschalidis [13] considered a model of a multi-class make-to-stock manufacturing system. The objective of their work is to derive a production policy that minimizes inventory cost subject to guaranteeing stock-out probabilities to stay bounded above by given constants, for each product class. Chen and Krass [14] investigated inventory models with minimal service constraints. They showed that the minimal service level constraint (SLC) model to be qualitatively different from their shortage cost counterparts and the transformation from an SLC model to a shortage cost model may not be always possible. de Kok [15] considered a lost-sales production/inventory control model with two adjustable production rates to meet demand. He obtained the practical approximations for optimal switch-over levels to such a model under service level constraints. Schneider [18] examined a  $(Q, s)$  model, he determined the optimal value of the order quantity  $Q$  and the reorder point  $s$  in which the average annual costs of inventory and orders are minimal under the condition that a certain service level is reached. Ouyang and Wu [19] developed an algorithm to find the optimal order quantity and optimal lead time for an inventory model with a service level constraint and when the probability distribution of the lead time demand is normal.

This paper extends the prior works [4,5,20] and studies the optimal lot-sizing for a production system with imperfect production and imperfect rework processes, random scrap rate, and under service level constraints.

## 2. Assumptions and notations

### 2.1. Assumptions

This research considers a realistic manufacturing system that may randomly produce  $x\%$  of defective items at a rate  $d$ . All items are screened and the inspection cost per item is included in the unit production cost  $c$ . The defective rate  $x$  is assumed to be a random variable with a known probability density function. A portion  $\theta$  of the defective items are scrap items, they are discarded before the rework process starts. Other portions  $(1 - \theta)$  of the imperfect quality items are assumed to be reworkable and when regular production ends, the rework process starts immediately at a rate  $P_1$ .

The rework process is assumed to be imperfect also, in that a portion  $\theta_1$  of the reworked items fail the repairing and become scrap items. The production rate  $P$  is assumed to be much larger than the demand rate  $\lambda$ . Hence, the production rate of defective items  $d$  can be expressed as  $d = Px$ . Let  $d_1$  denote the production rate of scrap items during the rework process, then  $d_1$  can be expressed as:  $d_1 = P_1\theta_1$ .

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