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A mixed-integer programming formulation for the general capacitated lot-sizing problem

Charles Sung, Christos T. Maravelias *

Department of Chemical and Biological Engineering, University of Wisconsin-Madison, Madison, WI 53706, USA

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Abstract

A new mixed-integer programming (MIP) formulation is presented for the production planning of single-stage multi-product processes. The problem is formulated as a multi-item capacitated lot-sizing problem in which (a) multiple items can be produced in each planning period, (b) sequence-independent set-ups can carry over from previous periods, (c) set-ups can cross over planning period boundaries, and (d) set-ups can be longer than one period. The formulation is extended to model time periods of non-uniform length, idle time, parallel units, families of products, backlogged demand, and lost sales.

Keywords: Production planning; Multi-item capacitated lot sizing; Mixed-integer programming; Set-up carry over; Set-up cross over

1. Introduction

To remain viable in today's highly competitive economy, chemical firms must use advanced planning methods to optimize their supply chains, from procurement and manufacturing to distribution and sales (Chopey, 2006; Grossmann, 2005). At the same time, product customization and diversification have led to larger numbers of final products, while the economic environment requires low inventories and higher utilization of existing units (Papageorgiou & Pantelides, 1996; Shobrys & White, 2002). Thus, different products are often produced in multi-product facilities, where limited resources are shared among competing tasks. Therefore, the economic impact of effective production planning methods can be significant. Furthermore, production planning is a hard optimization problem due to its combinatorial nature, and thus academically challenging.

The goal in production planning is to meet customer demand over a fixed time horizon divided into planning periods by optimizing the trade-off between economic objectives such as production cost and customer satisfaction level (Stadtler, 2005). The major decisions are production and inventory levels for each product in each planning period. To address production planning problems, research efforts have tried to adapt solution methods that have been successful for other applications. For some problems, however, the proposed approaches are insufficient because short-term decisions need to be taken into account to obtain good solutions. This can be accomplished by integrating production planning with detailed scheduling models. However, this leads to large optimization problems that are intractable for practical applications. Thus, despite all efforts that have gone into developing methods for the simultaneous production planning and scheduling of chemical plants, this remains a hard optimization problem (Crama, Pochet, & Wera, 2001; Kallrath, 2000; Pinto & Grossmann, 1998; Shapiro, 2004; Shah, 2005).

In this paper we develop a mixed-integer programming (MIP) formulation for the multi-item capacitated lot-sizing problem for a single processing unit. The proposed formulation overcomes several limitations of previous approaches. Most existing methods assume that if an item is produced in two consecutive periods, then it requires a set-up in each period. Furthermore, set-ups are assumed to begin and finish within the same time period and therefore are required to be shorter than that planning period. In the proposed formulation we relax these assumptions by allowing: (a) set-ups to carry over from previous periods, (b) set-ups to cross over planning period boundaries, and (c) set-ups to be longer than one planning period. By allowing set-ups to carry over, redundant set-ups are not required whenever the last item in a time period is the same as the first item in the

* Corresponding author. Tel.: +1 608 265 9026; fax: +1 608 262 5434.
E-mail address: maravelias@wisc.edu (C.T. Maravelias).

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Nomenclature

Indices

f	family, $f \in F = \{1, 2, \ldots NF\}$
i	product (state, item), $i \in I = \{1, 2, \ldots NI\}$
j	unit, $j \in J = \{1, 2, \ldots NJ\}$
t	planning period (time bucket), $t \in \{TB1, TB2, \ldots TBN\}$

Sets

AI	$i \in AI \subseteq I$ if production of i can be interrupted by idle time
I_f	$i \in I_f \subseteq I$ if i is in family f. Products in the same family use the same set-up

Parameters

b_i	backlog cost of product i
c_i	holding cost of product i
d_{it}	demand due of product i at end of time bucket t
d_{it}^{MIN}	minimum demand due of product i at end of time bucket t
h, h_t	length of planning period t
I_i^0	initial inventory of product i
r_i	production rate of product i
γ_i	set-up cost of product i
δ	small positive number
π_i	lost sales penalty for product i
τ_i	set-up time of product i

Continuous variables

$Cost$	objective function
I_{it}	inventory of product i at the end of time bucket t
$Idle_t$	amount of time in time bucket t in which no production occurs
$Late_t$	length by which ending boundary of time bucket t is delayed. Also is the length of set-up still uncompleted at the unmodified boundary
P_{it}	production level (amount) of i in time bucket t
S_{it}	number of set-ups for product i beginning in time bucket t
TSC_t	total cost of set-ups beginning in time bucket t
TST_t	total time of set-ups beginning in time bucket t

Continuous variables (extensions)

B_{it}	backlog of product i at the end of time bucket t
E_{it}	extra sales of product i in time bucket t
$Idle_{it}$	amount of time in time bucket t in state i in which no production occurs
L_{it}	lost sales of product i in time bucket t
P_{jit}	production level of i in time bucket t on unit j
TSC_{jt}	total cost of set-ups beginning in time bucket t on unit j

Binary variables

W_t	=1 if modified time bucket t is operated in SIP mode
X_{it}	=1 if i is the ending state of modified time bucket t
$\hat{X}_{it}$	=1 if set-up of product i crosses over the ending boundary of time bucket t
Y_{it}	=1 if modified time bucket t is operated in RP mode, starting and ending in state i
Z_{it}	=1 if state i is visited in modified time bucket t

following time period. By allowing set-ups to cross over period boundaries, set-ups may begin in one period and finish in a later period, thus better utilizing capacity. By allowing set-ups to be longer than planning periods, we gain the flexibility to discretize the planning horizon into periods of shorter and/or non-uniform length.

This paper is arranged as follows: In Section 2 we describe the production planning problem of interest, we discuss previously proposed methods, and we give the problem statement. In Section 3 we present the assumptions, basic concepts, and properties underlying our approach. In Sections 4 and 5 we present our mathematical formulation. In Section 6 we illustrate the applicability of the proposed approach through four example problems.

2. Background

2.1. Problem statement

In production planning we seek optimal decisions for production activities that transform raw materials into final products. Formally, we are given:

(i) A known planning horizon divided into N uniform or non-uniform time periods (i.e. time buckets), $t \in \{TB1, TB2, \ldots TBN\}$.
(ii) A set of products (items), $i \in \{A, B, \ldots\}$ with customer demand d_{it} due at the end of time period t and holding cost c_i.
(iii) Resource constraints: these may include unit and utility capacities, as well as raw material availability.
(iv) Production costs: these may include variable and fixed costs.

Optimization decisions are:

(i) the production level (amount) P_{it} of item i in period t,
(ii) the inventory level I_{it} of item i at the end of period t, and
(iii) the production cost TSC_t in period t.

The standard production planning problem assumes that it is possible for customer demand to be satisfied completely and on time. In this case, the objective is to fulfill customer demand at minimum total (i.e. production + inventory) cost. A network representation of the standard production planning problem is shown in Fig. 1. Inventory $I_{i,t-1}$ at the beginning of period t and