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An efficient MIP model for the capacitated lot-sizing and scheduling problem with sequence-dependent setups

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ABSTRACT

This paper presents a novel mathematical programming approach to the single-machine capacitated lot-sizing and scheduling problem with sequence-dependent setup times and setup costs. The approach is partly based on the earlier work of Haase and Kimms [2000. Lot sizing and scheduling with sequence-dependent setup costs and times and efficient rescheduling opportunities. International Journal of Production Economics 66(2), 159–169] which determines during pre-processing all item sequences that can appear in given time periods in optimal solutions. We introduce a new mixed-integer programming model in which binary variables indicate whether individual items are produced in a period, and parameters for this program are generated by a heuristic procedure in order to establish a tight formulation. Our model allows us to solve in reasonable time instances where the product of the number of items and number of time periods is at most 60–70. Compared to known optimal solution methods, it solves significantly larger problems, often with orders of magnitude speedup.

1. Introduction

This paper considers the lot-sizing and scheduling problem involving production of multiple items on a single finite capacity machine with sequence-dependent setup costs and setup times. In this problem, the decision maker must decide which items to produce in which periods, and must specify the exact production sequence and production quantities to satisfy deterministic dynamic demand over multiple periods that span a planning horizon, in order to minimise the sum of setup and inventory holding costs. The consideration of capacity limitations, significant sequence-dependent setup costs and non-zero setup times exacerbates the inherent difficulty in solving lot-sizing and scheduling problems and restricts the problem size that can be tackled in reasonable time. Ignoring these features when planning production aggravates costs and reduces productivity, particularly in process industries such as chemicals, drugs and pharmaceuticals, pulp and paper, food and beverage, textiles, or ceramics. Other examples include discrete manufacturing in industries such as aerospace, defense and automotive. All such manufacturers could benefit significantly from progress in this research area.

Recent work by Haase and Kimms (2000) proposes an exact optimisation approach to the problem. Their approach is based upon a mixed-integer programming (MIP) formulation. They start by generating all possible efficient sequences of items, and then use binary variables in the MIP to denote whether a sequence is selected for a given time period. However, the applicability of their approach is limited to either a small number of items or a

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short planning horizon. In this paper, we present an alternative model, which also uses pre-generated efficient sequences, but employs binary variables to indicate whether or not an item is produced in a given period. This yields smaller models, but makes it harder to express constraints on the setup costs. A naive formulation of these constraints gives loose LP relaxations, and hence an inefficient model. We then develop a heuristic algorithm which generates much tighter constraints.

We show experimentally that the proposed MIP model outperforms all previously known optimisation approaches to the capacitated lot-sizing and scheduling problem (CLSP) with sequence-dependent setups. It gives up to two orders of magnitude speedup in solution time over the Haase and Kimms model, and can solve larger instances. We also show that the efficient sequences can be generated more effectively, and that the same underlying model can be applied to a number of variants of the problem with similar time performance. The practical implications of these are significant.

The paper is organised as follows. In Section 2 we define the CLSP with sequence-dependent setups. Section 3 summarises previous work on this problem. In Section 4, we present an efficient dynamic program (DP) for generating the set of item sequences that might be applied in time periods in optimal solutions. Afterwards, we define a new MIP formulation of the problem (Section 5), and evaluate its performance on a set of randomly generated problem instances (Section 6). Finally, conclusions are drawn and directions of future research are outlined.

2. Problem definition

The *capacitated lot-sizing and scheduling problem with sequence-dependent setup times and costs* (CLSPSD) involves N_I different *items* able to be manufactured on a single machine over a series of N_T *time periods*. In each time period t, we must decide how many units x_t^i of each item i to produce. Since we have a single machine available, the production of different items within a time period must be sequenced. However, switching from item i to j requires a *setup*, which occupies $T^{i,j}$ units of the capacity in the given time period, and incurs $C^{i,j}$ cost. Producing one lot of item i employs the machine for $p^i x_t^i$ time, which is thus proportional to the lot size. The sum of all setup and production times within a time period cannot exceed the available *capacity* C_t in that period. The *demand* d_t^i for each item and time period is fully known in advance, and must be met exactly, either from production in that period or from excess produced in previous periods. The cost of a solution is composed of the sequence-dependent setup costs and the *inventory holding costs* h^i per excess unit of item i at the end of every time period.

The objective is to choose the production quantities and production sequences for each time period to meet the demand while minimising the total cost. The following assumptions are made.

(i) The cost of switching from item i to j can be computed as $C^{i,j} = q^i + rT^{i,j}$, where q^i is the *direct setup cost* of switching to item i, and r is the *time-proportional setup coefficient*.
(ii) Setup times satisfy the triangle inequality, i.e., $T^{i,j} \leqslant T^{i,k} + T^{k,j}$. Due to the previous assumption, the triangle inequality holds also for the setup costs.
(iii) The setup states are carried over from one time period to the next. It is allowed to switch from one item to another in idle periods (i.e., when no production occurs), but it incurs the same setup cost as if the item was produced.
(iv) Setups are performed within one time period. This also implies that a problem instance is feasible only if $T^{i,j} \leqslant C_t$ holds for all relevant pairs of items i and j and time period t.

In the micro-level representation of the solutions of CLSPSD, several items can be produced in each time period on the same machine, sequentially one after the other. Note that since setup times and costs are sequence-dependent, the sequence of item production in a period affects both feasibility and cost, and is a crucial issue for generating optimal solutions. Choosing a sequence of items $\sigma = (i_{k_1}, i_{k_2}, \ldots, i_{k_n})$ for production in time period t means that the machine is set up to produce item $\sigma[1] = i_{k_1}$ at the beginning of t; after producing a certain amount of $\sigma[1]$, a changeover from $\sigma[1]$ to $\sigma[2]$ occurs, and this continues until the end of time period t. At that point, the machine will be set up to produce item $\sigma[n_\sigma] = i_{k_n}$, where n_σ denotes the number items in sequence σ. Since setup states are carried over, the sequence applied in time period $t+1$ has to begin with item $\sigma[n_\sigma]$.

Note that applying sequence σ in t does not imply that a positive amount of items $\sigma[1]$ or $\sigma[n_\sigma]$ are actually produced in period t. It might happen that item $\sigma[1]$ was produced in period $t-1$, but switching from $\sigma[1]$ to $\sigma[2]$ takes place in t, or analogously, the machine is set up to item $\sigma[n_\sigma]$ so that period $t+1$ can start immediately with production. Hence, for the sake of simplicity, when saying an item is *produced* in a time period, we allow the production of zero quantities as well. At the same time, since the triangle inequality holds for the setup times, producing an empty lot of item $\sigma[k]$ for $k = 2, \ldots, n-1$ would lead to sub-optimality.

The micro-structure of a time period is illustrated in Fig. 1. Observe that the overall capacity required in time period t can be divided into two components. First, the capacity spent for setups, the amount of which depends only on the sequence applied, but not on the actual lot sizes. In contrast, the capacity required for production is proportional to the amounts of each item produced. The sum of these two components must not exceed the capacity available in the given time period.

3. Previous work on CLSPSD

Capacitated lot-sizing problems and their different variants are widely studied in the literature of operations research. A review of various lot-sizing and scheduling models, including small-bucket, large-bucket, and continuous time formulations is presented in Drexl and