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Non-cooperative strategies for production and shipments lot sizing in the vendor–buyer system

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A R T I C L E I N F O

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A B S T R A C T

This paper considers a decentralized dynamic production–distribution control. A discrete deterministic model in which a vendor produces a product and supplies it to the buyer is considered.

Several papers on vendor–buyer integrated production inventory management assume that policies are set by a central decision maker to optimize total system performance. Although vendor and buyer may agree to minimize the total cost, at least one of them has a private incentive to deviate from the agreement.

In the competitive situation, the objective is to determine schedules which minimize the individual average total cost of production, shipment and stockholding. We assume that the division of shipment costs is centrally coordinated or negotiated initially. It leads to a class of non-cooperative constrained games, indexed by two parameters connected with partitions of shipment costs. Non-cooperative strategies are considered as feasible strategies in a restricted non-cooperative game. Some properties of equilibrium strategies are investigated as acceptable equilibrium strategies of subgames in the game.

1. Introduction

One of the major tasks of supply chain management is to coordinate the processes in the supply chain in such a way that lowers system-wide cost is gained. In general, a supply chain is composed of independent partners with individual costs. For this reason, each firm is interested in minimizing its own cost independently. A well-integrated supply chain involves coordinating the flows of materials and information between distinct entities (as supplier, manufacturer, transporter, buyer, etc.). Both in the practice and in the literature considerable attention is paid to the importance of a coordinated relationships between entities in supply chain.

In the decentralized case, the power structure in vendor–buyer relationships as well as knowledge about the partner costs structure ought to be identified. Abad (1994) formulated the problem of buyer–vendor coordination as a two person cooperative game. Banerjee (1986a) and Goyal (1987) compute a price discount which compensates the loss of the buyer with respect to a cooperative policy. Kelle et al. (2003) also suggest some quantitative models to serve for motivation and a negotiating tool for providing joint operating policies. Sucky (2006) provides a bargaining model with asymmetric information in a dominated-buyer supply chain. He also describes the quantity losses due to a cooperative policy, both from the buyer's and the supplier's perspective. See also Viswanathan and Wang (2003) and Li et al. (2002) for leader–follower relationships under the Stackelberg game.

The idea of joint optimization for vendor and buyer was initiated by Goyal (1976) and Banerjee (1986b). A basic policy is any feasible policy where deliveries are made to the warehouse only when the warehouse has zero inventory. Several authors incorporated policies in

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which sizes of successive shipments from the vendor to the buyer within a production cycle either increases by a factor equal to the ratio of production rate to the demand rate or are equal in size. Neither the equal shipment size policy nor the increasing shipment size policy is always optimal. Hill (1999) combining these two policies derived the structure of a globally optimal production and shipment policy. All policies mentioned above are members of the following class of (k,n)-policies: *Initial* $k\geqslant 0$ *subbatches increasing in sizes* (*the vendor dominates the buyer*) *precede* $n\geqslant 0$ *subbatches equal in sizes* (*the buyer dominates the vendor*). These policies can also be viewed from competition perspective—as non-cooperative strategies for production distribution system.

Supply chain management has recently received a great deal of attention in the economy. It is natural that potential savings in cooperation (in centralized case) cannot be ignored. Competitive pressures drive profitability down. It may force firms to reduce costs while maintaining excellent customer service. A comprehensive literature review of works in this field is presented in Douglas and Griffin (1996) and Sarmah et al. (2006). Most studies on game theory models of supply chain (see Huang and Li, 2001) consider agents which maximize individual profit functions (with respect to purchase and sale prices). Bylka (2003) investigated equilibrium strategies in each (k,n)-non-cooperative game under the assumption that only the division of shipment cost (given by the pair (k,n)) is central coordinated or negotiated initially—before the game.

In most paper dealing with integrated inventory models (not only mentioned above), the transportation cost is considered only as a part of fixed setup or ordering cost. Ertogral et al. (2007) have studied the effects of incorporation of transportation cost into the model on possibility for better decision making under equal size shipment policies ($(0,n)$-policies). A fundamental advance in the two-side cost structure is in recognizing how delivery-transportation costs apply to both sides. David and Eben-Chaime (2003) suggested such a separation for $(0,n)$-policies. However, there is an additional set of problems involved in implementing (k,n)-policies (strategies) used in the decentralized (competed) case. The main issues are whether and how the vendor participates in the transportation cost in the case $k>0$.

The research presented in this paper offers game model without prices, where agents minimize individual costs. It is a non-cooperative game model of vendor–buyer competition in terms of the number and size of batches transferred between the two sides. It is a generalization of the paper Bylka (2003) with respect to the assumptions on the class of (k,n)-strategies. Additionally, an aspect of stability of equilibrium strategies is considered. The remainder of the paper is organized as follows. In Section 2, we develop the model describing inventory patterns and cost structure under (k,n)-strategies. It is then assumed that the players (the vendor and the buyer) compete for their size of batch decisions through a (k,n)-game. Nash equilibrium strategies are constructed in Section 3. However, the competition concerns only the shipments with exogenous number of transferred batches. This restriction is relaxed in Section 4.

2. Modelling vendor–buyer relationships under $(\boldsymbol{k},\boldsymbol{n})$-strategies

We consider a continuous deterministic model of a production–distribution system for a single product. A vendor produces a product on a single machine and supplies it to the buyer. Buyer's demand is a continuous function of the time. We denote

- P production rate of the vendor;
- D demand rate of the buyer;
- λ ratio P/D.

We examine the situation, where a vendor produces a product in a batch production environment and supplies it to the buyer under deterministic conditions (see Goyal, 1976). A schedule is determined by a sequence of cycles, each of them determined by

- Q the size of production batch;
- $m>0$ the number of shipments of subbatches per production cycle;
- (q_j,t_j) quantity and the moment of j-th shipment, $j=1,\ldots,m$.

Specifically, the problem will be characterized by the following assumptions:

A1. Constant production rate is sufficient to meet buyer's demand ($\lambda>1$) and buyer's demand must be satisfied.
A2. The final product is distributed by shipping it in discrete lots from the vendor's stock to buyer's stock (realized instantaneously).

In each cycle of the size Q there are some shipments which replenish the buyer's stock. The production starts at the moment, say 0, when the buyer have some initial inventories. A schedule $\tilde{q}=[(q_1,t_1),\ldots,(q_m,t_m)]$ determines the number, quantities and timing of shipments to the buyer. The production is stopped at the moment t^* and it starts again in the next cycle at T with the same buyer's initial inventories. We have

$$Q=\sum_{j=1}^{m}q_j,\quad T=\frac{Q}{D}\quad\text{and}\quad t^*=\frac{Q}{P}\left(=\frac{T}{\lambda}\right),\tag{1}$$

with the notation (for the vendor, $i=0$, and for the buyer, $i=1$):

- $q_0=I_1(0)$ the buyer's initial inventory position;
- $I_i(t)$ the inventory position at t just before the possible replenishment.

Therefore, for the initial cycle of the length T and $t\in[0,T]$, we have

$$I_0(t)=I_0(0)+P\min\{t,t^*\}-\sum\{q_j|t_j<t\},\quad j=1,\ldots,m,$$