Int. J. Production Economics 118 (2009) 19–25

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Lot sizing with learning, forgetting and entropy cost

M.Y. Jaber [a,]*, M. Bonney [b], I. Moualek [b]

[a] *Department of Mechanical and Industrial Engineering, Ryerson University, Toronto, ON, Canada M5B 2K3*
[b] *Nottingham University Business School, Jubilee Campus, Nottingham NG8 1BB, UK*

ARTICLE INFO

Article history:
Received 19 July 2006
Accepted 14 February 2007
Available online 20 August 2008

Keywords:
Lot sizing
Learning
Forgetting
Entropy cost

ABSTRACT

The lot sizing problem with learning and forgetting has received attention from many researchers. Most models in the literature adopt the classical approach of optimising the sum of two conflicting costs, which mainly are the holding and procurement costs. The results from these studies suggest that under learning effects, the optimal production policy is to deliver in smaller lots more frequently. Forgetting impedes performance and results in an optimal production policy that suggests having larger lots.

Some recent studies have suggested that it might be possible to improve production systems performance by applying the first and second laws of thermodynamics to reduce system entropy (or disorder). Including entropy cost suggested that larger quantities should be ordered than is suggested by the classical EOQ model. This paper investigates the lot sizing problem taking account of learning, forgetting and entropy cost. Learning encourages producing in smaller lots while forgetting and entropy cost encourage the use of larger lots. Mathematical models are developed and numerical examples illustrating the solution procedure are provided.

1. Introduction

The economic manufacture/order quantity (EMQ/EOQ), also known as the lot sizing problem, developed by Harris (1915) is believed to be the earliest published inventory management model and has been the cornerstone for the development of many inventory models since then. Among these models are those that incorporate the effects of learning and forgetting in production. This particular problem that was surveyed in Jaber and Bonney (1999) has been receiving considerable attention by researchers. This attention may be attributed to the importance of learning and forgetting in manufacturing environments that emphasise workforce flexibility where production is repetitive and usually in small batches (e.g., Wisner and Siferd, 1995).

The models surveyed in Jaber and Bonney (1999) that investigate the lot sizing problem (EMQ/EOQ) under learning and forgetting effects suggest an optimal production policy of delivering in smaller batch sizes, although they increase the control costs (entropy cost), smaller batch sizes can provide greater flexibility.

The EMQ/EOQ model is perhaps the most popular inventory model among academicians and practitioners (Osteryoung et al., 1986); however, it has been criticised by others. Selen and Wood (1987) cautioned that substantial miscalculations (or misinterpretations) of EOQ's input parameters often lead to poor results. Woolsey (1990) argued that EOQ assumptions are never met. Jones (1991) cautioned that most manufacturers who use the EOQ formula end up over estimating their lot sizes. This apparent problem may be attributed to the fact that the usual cost parameters used in the EOQ model, particularly ordering cost, holding cost and shortage cost, are difficult to estimate and so the results obtained may be misleading. Furthermore, estimating these costs becomes more important in a reverse logistics context

---

* Corresponding author. Tel.: +1 416 979 5000x7623; fax: +1 416 979 5265.
E-mail address: mjaber@ryerson.ca (M.Y. Jaber).

0925-5273/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2008.08.006

(Teunter, 2001). In addition to these costs there are hidden (or difficult to estimate) costs that are not usually accounted when modelling inventory systems. For example, excess managerial cost to control the improvement process (Ullmann, 1982), distribution costs (Pendlebury and Platford, 1988), cost of selling to customers—particularly when they demand less than a truck load (LTL) shipments, special labelling, direct delivery to retail outlets and last minute order changes (Gooley, 1995), increased labour union leverage and increased space requirements due to the rearrangement of machines into cells (Crusoe et al., 1999), warehouse personnel and racking costs (Fisher and Siburg, 2003), and price protection and product return costs (Callioni et al., 2005).

Some researchers have applied classical thermodynamics reasoning to modelling industrial engineering problems. For example, decision trees (Drechsler, 1968), manpower systems (Tyler, 1989), logistics management (Whewell, 1997), business process management (Chen, 1999), product life cycles (Tseng, 2004), inventory management (Jaber et al., 2004, 2006; Jaber, 2007), the price–quality relationship (Nuwayhid et al., 2006), co-ordinating orders in a supply chain (Jaber et al., 2006), and reverse logistics (Jaber and Rosen, 2008). Recently, King et al. (2006) implied that the first and second laws of thermodynamics can aid understanding of closed loop supply chains.

Jaber et al. (2004) postulated that the behaviour of production systems closely resembles that of physical systems. They suggested that improvements could be gained in production system performance by applying the first and second laws of thermodynamics to reduce system entropy. Jaber et al. (2004) introduced the concept of entropy costs to account for some of the hidden (or difficult to estimate) costs associated with production–inventory systems. To demonstrate the applicability of the first and second laws of thermodynamics to production systems, Jaber et al. (2004) used the EMQ/EOQ model as an illustrative example. The numerical results showed that the cost to control the commodity outflow (entropy cost) is higher for the classical model than for the suggested model probably reflecting the situation that large lots are cheaper to control than small ones. This observation falls in line with the findings of Cavinato (1991), who noted that small batch sizes congest distribution channels requiring tighter management and increasing the costs to monitor and manage the logistics operations. Some of these costs may be hidden or difficult to estimate. More examples of hidden costs associated with smaller batch size policies are found in Crawford et al. (1988) and Crusoe et al. (1999).

In-line with the above discussion, this paper investigates the lot sizing problem with learning, forgetting and entropy cost. This is done by incorporating the concept of entropy cost developed in Jaber et al. (2004) into the work of Jaber and Bonney (1998), who investigated the effects of learning and forgetting on the production lot size problem with infinite and finite planning horizons.

The remainder of this paper is organised as follows. Section 2 provides a brief introduction to the laws of thermodynamics, the concept of entropy cost and commodity flow strategies. Section 3 extends the classical economic manufacture quantity model by incorporating the entropy cost associated with the commodity flow strategy suggested in Section 2. Section 4 provides a brief description of the mathematics of the learning–forgetting process and the inventory model proposed by Jaber and Bonney (1998), and extends this model by accounting for entropy cost. Section 5 presents some numerical examples and discusses the results. Section 6 concludes the paper.

2. Thermodynamics and the concept of entropy

Jaber et al. (2004) postulated that a production system resembles a physical system operating within its surroundings, which for the production system include the market and the supply system. A physical thermodynamic system is defined by its temperature, volume, pressure and chemical composition. A system is in equilibrium when these variables have the same value at all points. In a similar manner, a production system could be described by its characteristics, for example the price (P) that the system ascribes to a certain commodity (or collection of commodities) that it produces.

Reducing the price of a commodity below the market price may increase customers' demand, and produce a commodity flow (sales) from the system to its surroundings. This could be considered to be similar to the flow of heat from a high-temperature reservoir (source) to a low-temperature reservoir (sink) in a thermodynamic system, where part of this heat is converted into useful work and some of the heat is expelled by the system and wasted. As shown in Fig. 1, some of the commodity flows to the market as sales and is converted to revenue (equivalent to useful work) and some is wasted (hidden cost).

To guarantee a flow of commodity from the inventory system to the market, the following strategies are considered. The first strategy suggests that a firm may provide the same quality product as its competitors at a lower price, and the second strategy suggests that a firm provides a better quality product than its competitors at the same price. These two strategies correspond with those suggested in the literature. For example, a supplier that seeks to improve its share of the market will attempt to increase customer satisfaction or sustain it relative to competitors, and/or reduce costs associated with providing the same level of satisfaction (Whewell, 1997, p. 18). Furthermore, Whewell (1997) advocates that changes to

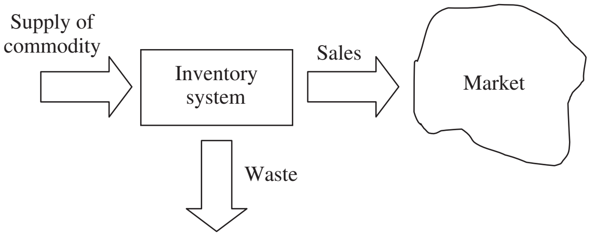

Fig. 1. The inventory system and its surrounding.