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Lot sizing with random yield and different qualities

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ABSTRACT

This paper considers a production/inventory system where items produced/purchased are of different qualities: Types A and B. Type A items are of perfect quality, and Type B items are of imperfect quality; but not necessarily defective; and have a lower selling price. The percentage of Type A (the yield rate) is assumed to be a random variable with known probability distribution. The electronics industry gives good examples of such situations. We extend the classical single period (newsvendor) and the economic order quantity (EOQ) models by accounting for random supply and for imperfect quality (Type B) items which are assumed to have their own demand and cost structure. We develop mathematical models and prove concavity of the expected profit function for both situations. We also present detailed analysis and numerical results. We focus on comparing the profitability of the novel proposed models with models from the literature (and derivatives of these models) that develop the optimal order quantity based on the properties of Type A items only (and ignore Type B items). We find that accounting for Type B items can significantly improve profitability.

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1. Introduction

A typical assumption in conventional inventory models is that ordered/produced items are of perfect quality. In reality, and due to limitation of quality control procedures, among other factors, items of imperfect quality are often present (see, for example, Yano and Lee [1]). This fact has been recognized by many researchers who study "random yield" models considering uncertainty in the supply process due to quality problems. A typical assumption in this literature is that defective units are reworked or removed from inventory. In some situations, imperfect items cannot be reworked or returned to the supplier. It may then be possible to sell items of imperfect quality at a low price. This paper considers such a situation.

Specifically, in this paper we consider two classical inventory models: The single period (newsvendor) and the basic economic order quantity (EOQ) models. For each of these two models, we assume that the supply process is characterized by a random "yield rate" with a known probability distribution. That is, the amount received from each order contains a proportion of perfect quality (Type A) items with a known probability distribution. The remaining proportion is of imperfect quality (Type B) items. We assume that each of Type A and B items have their own demand and cost structure (while sharing the same supply source). We formulate mathematical models that determine the optimal lot size in a way that maximizes the expected profit from both Type A and B items. We then prove concavity of the expected profit and present somewhat simple optimality conditions for the newsvendor model and a closed-form expression for the optimal lot size for the EOQ model. We also compare the profitability of our models with models from the literature that determine the optimal lot size based on the characteristics of Type A items only. The main contribution of this paper is accounting for imperfect quality (Type B) items in determining the optimal lot size under random yield, and showing that doing so can significantly improve profitability.

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This paper is motivated, in part, by our experience with the air-conditioning "split" units market in Lebanon. We observed that the prices of split units of identical characteristics (and sometimes with the same brand name) are different between specialized electronics stores and major department stores, with the prices at the later being lower. An engineer owning a small firm working with the installation of air-conditioning units explained this difference in pricing to us. One of the reasons for this pricing variation is that the split units sold at major department store sometimes have minor defects (e.g., a problem with the plastic utilized for the external frame). The engineer further explained that some major air-conditioning manufacturers serving the Lebanese market re-label these "defective" products with some "no-name" brands and sell them at low prices, while other manufacturers keep their brand names on these items. Thus, it seems that air-conditioning manufacturers serving Lebanon have a demand and cost structure based on producing and selling perfect and imperfect items similar to the model in this paper.

We also observe potential retail applications for our model. One retailer in an upper-class Lebanese neighborhood, importing Christmas decorations from the Far East, has indicated that he is often aware that the shipments he receives contain imperfect quality items. The retailer mentioned that he tolerates these imperfections due to suppliers offering price discounts from and guarantees that the fraction of imperfect items will not exceed a certain limit. In addition, the retailer reported that upon screening and identifying imperfect quality items, these items are salvaged to another retailer who sells them in a secondary market at a lower price. The model we develop in this paper could serve to determine the ordering policy that maximizes the joint profits of both retailers. Some profit-sharing schemes could be devised to share the benefits of the centralized ordering policy as it is done in supply chain contexts (see, for example, Banerjee [2]). However, for the sake of simplicity and ease of presentation, we exclude such profit-sharing schemes from our model in this paper.

Literature review. Related literature is on inventory models with random yield/imperfect quality. Yano and Lee [1] and Wright and Mehrez [3] present detailed reviews of this literature. Most related are works with inventory models under random yield within the newsvendor and EOQ setting. Khouja [4] provides a review of the works that deals with the newsvendor model under random yield. In the following, we present a brief review of newsvendor- and EOQ-type models under random yield. We focus on works that are closely related to the models in this paper. Interested readers are referred to [4,1,3] for exhaustive reviews.

Motivated by agricultural applications, Karlin [5] is the first to study the newsvendor model under random supply. He assumes that the amount received from an order is a random variable with a known probability distribution and considers the case where initial inventory may be available at the beginning of the selling period. He proves that the optimal ordering policy is a simple order-up to policy under general conditions on the cost and demand parameters. Shih [6] considers a model similar to that of Karlin [5] within a production context. He assumes that the proportion of defective items is random with a known probability distribution and defective items are returned to the manufacturer at no cost. Shih [6] does not consider initial inventory and utilizes a linear cost structure. He proves the convexity of the expected cost function and derives a simple critical-fractile type optimality condition that gives the optimal lot size. The single period newsvendor model we develop in this paper may be seen as an extension of the work of Shih [6] by accounting for imperfect (Type B) items in determining the optimal lot size.

Noori and Keller [7] consider a newsvendor problem similar to that of Shih [6] and develop specialized techniques for determining the optimal lot size for exponential and uniform demand distributions. Following Silver [8], Noori and Keller consider two cases: (i) the standard deviation of the amount received from an order is independent of the order size (which is equivalent to the assumptions of Karlin [5] and Shih [6]); and (ii) the standard deviation of the amount received is proportional to the order size. Additional references on the newsvendor model with random yield include Ehrhardt and Taube [9], Gerchak et al. [10], and Henig and Gerchak [11].

The standard EOQ model under random supply was first studied by Silver [8]. As aforementioned, he considers two cases of supply randomness involving a variance of the amount received from an order being either independent or proportional to the order size. Silver [8] derives closed-form expressions for the optimal order quantity in both cases involving the first two moments of the amount received. He argues numerically that utilizing the classical EOQ order quantity will result in a low penalty in terms of increasing the expected cost for a wide range of cases. Shih [6] also develops an EOQ model under the assumption that the proportion of imperfect quality items in an order has a known distribution. He derives an expression for the optimal lot size similar to that of Silver [8] (under the assumption that yield variability is independent of the lot size). Our EOQ model in this paper may be seen as an extension of the works of Shih [6] and Silver [8] by accounting for imperfect quality (Type B) items in determining the optimal lot size.

Other related works on the EOQ model with random yield include Kalro and Gohil [12] and Mak [13] who extend Shih [6] and Silver [8] models by allowing for shortages. In addition, Porteus [14] considers an EOQ model with a Markovian for a production process that starts "in-control" when producing a lot. Then, the process can go out of control with a constant probability upon producing each unit of the lot. This leads to a proportion of defective items which increases with the lot size, and consequently to a smaller optimal lot size (as compared to the classical EOQ model). Porteus [14] assumes that defective items are reworked instantaneously and returned to inventory at a cost. Rosenblatt and Lee [15] consider a model similar to that of Porteus [14] within the context of the economic production quantity (EPQ) model but assume that the time until the process goes out of control is exponentially distributed. In a recent paper, Freimer et al. [16] claim to develop a model for the yield rate (within an EPQ context) which generalizes the approaches of Shih [6], Porteus [14], and Rosenblatt and Lee [15], among others. However, Freimer et al. ignore the variance of the yield rate in their model without a reasonable justification.

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