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Multiple lot-sizing decisions with an interrupted geometric yield and variable production time

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ABSTRACT

This study examines a multiple lot-sizing problem for a single-stage production system with an interrupted geometric distribution, which is distinguished in involving variable production lead-time. In a finite number of setups, this study determined the optimal lot-size for each period that minimizes total expected cost. The following cost items are considered in optimum lot-sizing decisions: setup cost, variable production cost, inventory holding cost, and shortage cost. A dynamic programming model is formulated in which the duration between current time and due date is a stage variable, and remaining demand and work-in-process status are state variables. This study then presents an algorithm for solving the dynamic programming problem. Additionally, this study examines how total expected costs of optimal lot-sizing decisions vary when parameters are changed. Numerical results show that the optimum lot-size as a function of demand is not always monotonic.

1. Introduction

Multiple lot-sizing production-to-order (MLPO) problems have been studied for several decades (Bowman, 1955). Such problems typically arise from variations in production yield. Consider a production system with an uncertain process yield. To fulfill a particular customer demand, lots may need to be released several times to minimize total expected costs. The MLPO problem is to determine the optimal lot-size for each possible lot release.

This study describes and formulates a single-stage MLPO problem with one salient feature—uncertain production lead-time. According to Yano (1987), this feature may arise due to many factors such as unreliable vendors, unreliable transportation time, job queuing, machine breakdowns, and rework. Uncertain lead-time characteristic has seldom been considered in MLPO studies; although it has been examined in production control studies (Hsu, Wee, & Teng, 2007). In this study, we assume production lead-time is a random variable; the probability for one period is p and that for two periods is $1 - p$.

In the MLPO problem, process yield follows an interrupted geometric (IG) distribution. The delivery agreement includes due dates; that is, customers will not accept products after delivery due dates, and salvage values of products are negligible. In contrast, finished goods produced ahead of the due date become inventory and incur holding costs. The following cost items are included: setup cost, variable production cost, inventory holding cost, and shortage cost.

An example of the MLPO problem in this study is a process of drawing special steel coils. The manufacturing process has two operations: pickling and wire drawing. The pickling operation removes rust from steel coils. The processing time required for pickling a steel coil varies. In practice, a steel coil undergoes one or two pickling operations depending on the duration the coil has been in air. The drawing operation reduces the size of the input coil. Drawing speed is very fast. All coils in a lot are inspected when the whole lot is complete. The drawing operation involves a die that is worn gradually over time. When this die is excessively worn, the output does not meet specifications. This implies that the integrated drawing process follows an IG distribution, and production lead-time for a lot from release to output takes one or two periods. Special steels are customized products that in most cases cannot be sold to other customers. Thus, we assume product salvage value is negligible.

This study develops a dynamic programming (DP) approach to solve the MLPO problem. Several lemmas are proposed to reduce the DP problem solution space. Numerical experiments show that the optimum lot-size, as a function of demand, is not necessarily monotonic. This study experimentally investigated how total expected costs of optimal lot-sizing decisions vary when various parameters change.

The remainder of this paper is organized as follows. A literature review is given in Section 2. Section 3 presents the MLPO problem as a DP model by including a simple example to facilitate understanding the formulation. Lemmas for reducing the DP solution

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Notation

D	quantity required by a customer
T	number of periods in the decision time horizon
t	index of time, $t = 0$ is the due date, $t = 0, 1, 2, \ldots, T$
α	setup cost incurred at each lot input, $\alpha > 0$
β	variable production cost per unit, $\beta > 0$
k_t	lot-size released at t
W_t	a binary variable indicating the demand of a setup, $W_t = \begin{cases} 0 & if \quad k_t = 0 \\ 1 & if \quad k_t > 0 \end{cases}$
D_t	remaining demand at t (number of demand units still not fulfilled at t)
h	inventory holding cost per unit per period ($/unit-period), $h > 0$
m	shortage cost per unit, $m > 0$
p	probability of producing a lot in one period
$1 - p$	probability of producing a lot in two periods
θ	probability that the production system is in-control
Y_{k_t}	a random variable for the number of output units for lot k_t
$R_t(k_{t+1})$	number of work-in-process (WIP) at t, $R_t(k_{t+1}) = \begin{cases} 0 & \text{if the realized production time for } k_{t+1} \text{ is one period,} \\ k_{t+1} & \text{otherwise} \end{cases}$
$s_t = (D_t, R_t(k_{t+1}))$	the production system status at t, also called *state t*
$C_t(s_t, k_t)$	total expected cost incurred after t
$C_t^*(s_t) = \underset{0 \leqslant k_t \leqslant \infty}{Min} \{C_t(s_t, k_t)\}$	minimum total expected cost incurred after t
$N_t(s_t)$	optimal lot-size at state s_t; that is, $\underset{0 \leqslant k_t \leqslant \infty}{Min} \{C_t(s_t, k_t)\} = C_t(s_t, N_t(s_t))$

space are presented in Section 4. An algorithm for solving the DP is presented in Section 5. Numerical examples are given in Section 6. Conclusions are provided in Section 7.

2. Related literature

Grosfeld-Nir and Gerchak (2004) and Yano and Lee (1995) comprehensively surveyed studies of MLPO problem. Such studies can be categorized as: *single-stage* and *multiple-stage*. This study is in the category of single-stage MLPO problems; thus, recent studies in this category are reviewed.

Recent single-stage MLPO studies can be analyzed from multiple perspectives. The first perspective is associated with customer demand and delivery requirements. Customer demand may be *stochastic* (Gerchak & Grosfeld-Nir, 1998) or *deterministic*. Delivery requirements can be based on due dates or quantities. In a quantity-based agreement (also called *rigid-demand delivery*), the quantity ordered must delivered in full; that is, partial delivery is unacceptable. In due-date-based agreements (also called *non-rigid demand delivery*), customers will not accept products after the due date. Prior studies are either based on rigid-demand (e.g., Anily, 1995; Anily, Beja, & Mendel, 2002; Beja, 1977; Zhang & Guu, 1998), or non-rigid demand (e.g., Guu & Zhang, 2003; Pentico, 1988; Sepehri, Silver, & New, 1986; Wang & Gerchak, 2000).

The second perspective is associated with production characteristics such as process yield, lead-time, and quality classifications. Previous studies assumed process yield is governed by a probability distribution, which includes the discrete uniform (Anily, 1995), the binomial distribution (Beja, 1977; Pentico, 1988; Sepehri et al., 1986), the interrupted geometric (Anily et al., 2002; Guu & Zhang, 2003; Zhang & Guu, 1998), the general distribution (Zhang & Guu, 1997), and the stochastically proportional (Grosfeld-Nir & Gerchak, 1990; Wang & Gerchak, 2000). In terms of lead-time, few studies (Wang & Gerchak, 2000) addressed an MLPO problem in which production lead-time is longer than the time epochs between any two lot releases. Most reseachers assumed production outcomes have only two possible states, either acceptable or unacceptable quality, while a few other studies (Gerchak & Grosfeld-Nir, 1999) examined scenarios that may have three or more outcomes—for example, high quality, medium quality and unacceptable quality.

The third perspective is associated with cost items and objective functions for the MLPO decision making. The most widely addressed cost items include setup cost, variable production cost, inventory cost, and shortage cost. A few researchers also considered inspection cost (Grosfeld-Nir, Gerchak, & He, 2000) and disposal cost (Wang & Gerchak, 2000). For the objective function, most researchers attempted to minimize total expected cost, while a few considered the impact of risk caused by cost variance (Grosfeld-Nir & Gerchak, 1996).

The fourth perspective is associated with the solution approach. Most formulations of MLPO problems include recursive formulas and have been widely interpreted as DP problems. Therefore, DP has been widely used to solve MLPO problems; however, such a solution approach may be very demanding computationally. Some researchers proposed lemmas to reduce the solution space (Anily, 1995; Beja, 1977; Zhang & Guu, 1998); some others attempted to develop near–optimal heuristic rules (Pentico, 1988; Sepehri et al., 1986); and a few others approximately model the DP problem using a relatively simpler non-DP problem for cases with extremely large/small demand quantities (Anily et al., 2002).

The four perspectives highlight the various complex scenarios that can occur in single-stage MLPO problems. Some researchers investigated multiple-stage MLPO problems, in which additional complexity may arise due to inclusion of lot-sizing decisions made at the start of each stage. For example, a production system with two stages needs a lot-sizing decision for the first stage. Releasing all output items of the first stage immediately to the next stage may not be an optimal decision. A lot-sizing decision at the start of the second stage is needed. Example studies that addressed the multiple-stage MLPO problem include Grosfeld-Nir (2005) and Grosfeld-Nir and Robinson (1995).

Compared to those in literature, the singe-stage MLPO problem in this study is unique in that it includes one salient feature—*uncertain production lead-time*. This feature has rarely been considered in either single-stage or multiple-stage MLPO studies.

3. Modeling

To model the MLPO problem, the notation is first presented, followed by a description of the IG distribution. A simple example is then given to explain the idea of the formulation. Finally, the cost function of the MLPO problem is modeled using a recursive formula, and its boundary conditions (BCs) are defined.

3.1. IG distribution

As process yield is governed by an IG distribution, the production system manufactures each unit in a one-by-one manner and operates in two possible states *in-control* or *out-of-control*. The output unit is non-defective when the system is in-control, and is defective when the system is out-of-control. The process can