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A fix-and-optimize approach for the multi-level capacitated lot sizing problem

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ABSTRACT

This paper presents an optimization-based solution approach for the dynamic multi-level capacitated lot sizing problem (MLCLSP) with positive lead times. The key idea is to solve a series of mixed-integer programs in an iterative fix-and-optimize algorithm. Each of these programs is optimized over all real-valued variables, but only a small subset of binary setup variables. The remaining binary setup variables are tentatively fixed to values determined in previous iterations. The resulting algorithm is transparent, flexible, accurate and relatively fast. Its solution quality outperforms those of the approaches by Tempelmeier/Derstroff and by Stadtler.

1. Introduction

This paper treats the problem to determine time-phased production quantities (lot sizes) for multi-level production systems in which a changeover at a resource from one product type to another requires a setup time and/or causes setup cost. The capacity of the resources is limited. The time-phased demand is assumed to be given and has to be satisfied. To date, this type of lot sizing problem cannot be solved satisfactorily within computerized Material Requirements Planning (MRP) modules of Enterprise Resource Planning (ERP) systems. The reason is that these systems often ignore the capacity limits of the production system while computing the lot sizes. This leads to infeasible production schedules which result in long and unpredictable lead times and large in-process inventories.

The problem of finding capacity-feasible production quantities for a multi-stage production system that minimize setup and holding cost has been formally stated as the multi-level capacitated lot sizing problem (MLCLSP), see Billington et al. (1983). For capacitated production systems with non-zero setup times, the question whether a feasible production plan (without overtime) exists has shown to be already $\mathcal{NP}$-complete, see Maes et al. (1991). Several authors have therefore developed heuristics for the problem. Multi-level capacitated lot sizing is hence both practically important and scientifically challenging.

From a practical point of view, generic lot sizing approaches for ERP systems should meet several requirements. First, they should be adaptable to specific aspects of a concrete production system. This favors flexible approaches based on mathematical programming as opposed to “hard-wired” procedural heuristics that follow a highly problem-specific logic. Second, they should enable the planner to find feasible solutions of acceptable quality quickly and then let him decide to invest additional computation time to improve the solution. Third, if lot sizing on the one hand and sequencing/scheduling on the other hand are treated separately, it should always be possible to disaggregate the (aggregated) lot sizes into a detailed production schedule in continuous time. In a multi-level production system, this can only be guaranteed if positive lead times between the

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production stages are considered. Assume a serial product structure with one end product that requires a single component. Both items are produced on different resources. Furthermore, assume that the resource for the end product is fully loaded to produce a single unit of the end product in a period. In this case, it is not feasible to produce a unit of the component in the same period so that it enters the unit of the end product at the beginning of the same period. Therefore, this unit of the component must be produced at least one period ahead. For this reason, a positive lead time is required.

Reviews of the literature on dynamic lot sizing are given by Bahl et al. (1987), Maes and van Wassenhove (1988), Gupta and Keung (1990), Salomon et al. (1991) and Buschkühl et al. (2008). Kuik et al. (1994) relate lot sizing to batching and comment on some general criticism of batching analysis. The progress with single-item lot sizing is analyzed by Wolsey (1995) and Brahimi et al. (2006). Drexl and Kimms (1997) discuss models that consider both lot sizing and scheduling. Karimi et al. (2003) give a review of solution approaches for single-stage capacitated lot sizing problems and Jans and Degraeve (2007) focus on metaheuristics for dynamic lot sizing. Quadt and Kuhn (2008) review capacitated lotsizing problems with extensions. Table 1 classifies papers on the MLCLSP by the general solution approach, i.e. mathematical programming-based approaches, Lagrangean relaxation and decomposition, decomposition and aggregation, local search and metaheuristics and finally greedy heuristics. Some of these papers are discussed below in more detail.

In Tempelmeier and Helber (1994), Helber (1995) and Tempelmeier and Derstroff (1996), early product-oriented decomposition approaches for the MLCLSP are presented. The Tempelmeier/Destroff-heuristic (TDH) is until now the fastest available method for the MLCLSP. It is based on a Lagrangean relaxation of the MLCLSP which then decomposes into single-product uncapacitated lot sizing problems of the type studied by Wagner and Whitin (1958). The Lagrangean relaxation leads to a lower bound on the objective function value. A heuristic finite scheduling procedure is used to create a feasible solution and to compute an upper bound on the optimal objective function value. While the algorithm is fast, it is difficult to describe and implement as well as inflexible with respect to modifications of the underlying problem. The solution quality especially for large problem instances offers opportunities for improvement. Katok et al. (1998) present a linear-programming (LP)-based approach that works with a heuristic modification of the coefficients of the production quantities in both the objective function and the constraints. Tempelmeier (2008, p. 342) shows that this concept is very vulnerable when setup times occur and capacity limits are tight. In these situations existing feasible solutions (without overtime) are not found.

Table 1
Literature on the MLCLSP.

Mathematical programming approaches
Billington et al. (1986), Maes et al. (1991), Pochet and Wolsey (1991), Kuik et al. (1993), Clark and Armentano (1995), Harrison and Lewis (1996), Stadtler (1996), Stadtler (1997), Katok et al. (1998), Belvaux and Wolsey (2000, 2001), Rossi (2005), Stadtler (2003), Sürie and Stadtler (2003)
Lagrangean relaxation and decomposition
Tempelmeier and Derstroff (1993, 1996), Özdamar and Barbarosoglu (1999, 2000), Moorkanat (2000), Chen and Chu (2003)
Decomposition and aggregation
Tempelmeier and Helber (1994), Helber (1994), Boctor and Poulin (2005)
Local search and metaheuristics
Salomon et al. (1993), Kuik et al. (1993), Helber (1994, 1995), Barbarosoglu and Özdamar (2000), Hung and Chien (2000), Özdamar and Barbarosoglu (2000), Özdamar and Bozyel (2000), Gutierrez et al. (2001), Xie and Dong (2002), Berretta and Rodrigues (2004), Berretta et al. (2005), Pitakaso et al. (2006)
Greedy heuristics
Clark and Armentano (1995), França et al. (1997)

Stadtler (2003) proposes a mixed-integer programming-based heuristic that solves a series of subproblems through internally rolling schedules with time windows. For the periods within the time window of a particular subproblem, the simple plant location variant of the lot sizing problem is used to speed up the optimization process. Stadtler's approach delivers high-quality solutions for problems with zero lead times, but cannot deal with positive lead times (Stadtler, 2003, p. 501). The reason is that in a general product structure multiple relations with different lead times between end product quantities and time-phased resource requirements can occur. Hence, this makes a consistent disaggregation into a detailed production schedule in continuous time impossible. In addition, solution times for his approach increase substantially as the problem size (number of binary setup variables) increases. A variant of Stadtler's general approach of internally rolling schedules for the Capacitated Lot Sizing Problem with Linked Lot Sizes (Haase, 1994, 1998) is presented by Sürie and Stadtler (2003). It is based on an extended model formulation and valid inequalities to yield a tight formulation that speeds up the branch&bound process.

Belvaux and Wolsey (2001) show how to develop tight formulations for several special problem features occurring in practice. Rossi (2005) develops a time-oriented decomposition similar to the one by Stadtler where some of the setup variables are initially relaxed while others are iteratively fixed. Unfortunately, the author provides no direct comparison to the procedures by Stadtler and by Tempelmeier. Pitakaso et al. (2006) present a decomposition algorithm for the MLCLSP based on a limited subset of products and periods. Each problem in the decomposition is solved to optimality and a capacity reservation mechanism is used to reflect products and periods "outside" of the current problem. The decomposition itself is controlled by an "ant colony optimization" algorithm. The computation times that are necessary to find better results than with Stadtler's method appear to be quite high (20–30 min) and then the average improvement is