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Lot sizing with a Markov production process and imperfect items scrapped

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ABSTRACT

This paper considers an inventory system within the economic order quantity (EOQ) model framework under random supply. The production process can go "out-of-control" with a constant probability and start producing imperfect quality items. Such a system has been studied in the literature under the assumption that defective items are reworked instantaneously and returned to inventory. In some situations, imperfect quality items are not necessarily defective and are removed from the inventory to be used elsewhere. This paper develops two models where imperfect items are removed from inventory. The first model finds the expected cost and optimal lot size assuming that imperfect quality items are removed from inventory at no cost. The second model assumes that batches of imperfect quality are consolidated and shipped together due to economies of scale in shipping. Analytical and numerical results are developed for both models revealing several managerial insights.

1. Introduction

The effect of supply uncertainty on inventory control has been recognized by many researchers. Several research papers developed inventory models where the quantity received may not match the quantity ordered due to an unreliable supply process. Such an unreliable process may be due to a production system yielding imperfect quality items. Wright and Mehrez (1998) and Yano and Lee (1995) present a detailed survey of inventory models with random supply and imperfect quality output.

The research on inventory systems is based on different assumptions concerning the underlying process generating imperfect quality and the way imperfect quality items are handled. One stream of research assumes that the proportion of imperfect quality items in a lot is random with a known probability distribution independent of the lot size and that defective items are removed from inventory. This research can be traced back to the papers of Karlin (1958), Shih (1980), and Silver (1976). Recent research in this area includes the work of Salameh and Jaber (2000) who incorporate some cost structure of imperfect quality items by assuming that identifying imperfect quality items requires a screening process initiated upon the receipt of an order. Imperfect quality items are then sold as a single lot for a reduced price at the end of the screening process. Related to this work is the paper by Cárdenas-Barrón (2000) where an error appearing in Salameh and Jaber's (2000) paper is corrected. The error did not affect the main idea and the remainder of the paper of Salameh and Jaber (2000). Several authors recently adopt Salameh and Jaber's assumptions on handling of imperfect quality (e.g., Goyal and Cárdenas-Barrón, 2002; Chang, 2004; Huang, 2004; Papachristos and Konstantaras, 2006; Maddah and Jaber, 2008). In particular, Maddah and Jaber (2008) rectify a flow in Salameh and Jaber's model by properly applying renewal theory and extend the analysis by assuming that batches of imperfect quality are consolidated and shipped jointly. In a more recent work, Maddah et al. (2009) assume that imperfect quality items have their own demand and cost structure and develop integrated policies that account for these items.

Another stream of research assumes a Markov production process that may go out-of-control at a certain time, leading to some or all items produced from that time onwards to be of imperfect quality. This research originated with Porteus (1986) and Rosenblatt and Lee (1986). Porteus (1986) considers an EOQ model and assumes that the production process can go out-of-control with a constant probability every time an item is produced, and once the process is out-of-control, it remains at that state with all items produced beyond that point being of imperfect quality. This assumption is supported by the fact that "once most production processes drift out of the basic variance range they keep on doing it usually for a reason that can be determined and corrected" (Hall, 1983). Porteus (1986) develops an approximate model giving the total cost and optimal lot size. Rosenblatt and Lee (1986) consider an economic production quantity (EPQ) model with a finite production rate and assume

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that the time until the process goes out-of-control is exponentially distributed. In a recent paper, Freimer et al. (2006) extend the work of Rosenblatt and Lee (1986) by considering that the fraction of items produced is a general increasing function of time, rather than depending on a specific distribution of the time until the process goes out-of-control.

The previous studies of Porteus (1986) and Rosenblatt and Lee (1986) assume that defective items are reworked instantly and returned to inventory at a cost. These assumptions do not always hold in practice, as it may not be possible to repair certain types of defective items. In fact, Porteus (1986) states that the assumption of instant rework is "for simplicity." For example, the imperfect quality items in some electronic and clothing industries are not necessarily defective and are not reworked. Rather, they may be discarded, for example, to be used in another production/inventory situation, or sold at a discounted price. Maddah and Jaber (2008) present two real-life examples on air conditioning split units and Christmas decorations, where imperfect items are not reworked, but rather sold at a discount price in a secondary market.

In this paper, we consider an EOQ model with a two-state Markovian production process (as in Porteus, 1986) where, unlike Porteus (1986), imperfect quality items are not necessarily defective and they cannot be reworked. Specifically, this paper assumes that imperfect quality items are removed from inventory at the end of a lot production to be either discarded or shipped to a different market. As discussed previously, this assumption has been adopted by other researchers (e.g. Karlin, 1958; Silver, 1976; Shih, 1980; Salameh and Jaber, 2000; Papachristos and Konstantaras, 2006) and applies to many production and inventory situations. However, to the best of the authors' knowledge, there is no work in the literature that considers a Markovian production process with this assumption. This paper therefore contributes a natural extension of the existing literature allowing for a more versatile range of applications.

In addition, this paper also addresses the critical issue of *when* to ship the imperfect quality items, through a refined model. This is another contribution of this paper. This model includes a cost structure for imperfect quality items by assuming that batches of these items resulting from different production lots can be consolidated and shipped jointly. This is sought to balance the shipping (perhaps disposal) and holding costs of imperfect quality items with the familiar production and holding costs of perfect items. This model is in the same spirit of its counterpart in Maddah and Jaber (2008). However, the supply process is different here, as Maddah and Jaber (2008) assume that the percentage of quality items is a random variable which is independent of the lot size.

Since our analysis approach and many of our results are similar to Porteus (1986), we clarify, in the following, the contribution of this paper relative to the work of Porteus (1986). As aforementioned, the main difference between this paper and Porteus (1986) is the way imperfect quality items are handled. Porteus (1986) assumes that imperfect items are reworked instantaneously at a cost, while this paper assumes that imperfect quality items are removed from inventory. The assumption of instantaneous rework in Porteus (1986) simplifies the analysis as production cycles are completely identical. Ordering and holding costs can be found similar to those of the classical EOQ model. The main complication brought by quality considerations in Porteus (1986) is the rework cost which is proportional to the mean number of imperfect items in a production lot. In this paper, with the assumption that imperfect items are removed from inventory, the production cycles are no longer identical (rather renewable points are defined at the beginning of every cycle), and the expected cost depends on both the mean and variance (second moment) of the amount of imperfect quality items in a lot. One contribution of this paper is deriving an expression for the variance of the number of imperfect quality items resulting from a two-state Markov process and incorporating this in the expected cost function.

The remainder of this paper is organized as follows. Section 2 describes the mathematical model and assumption and presents an exact derivation of the expected cost function. Section 2 also discusses the convexity of the expected cost function. Section 3 develops an approximate model which leads to a closed-form solution of the optimal order quantity and several analytical results. Section 4 refines the initial model of Section 2 by considering the cost resulting from consolidation and shipping of imperfect quality items. Section 5 includes numerical results for both the exact and approximate models that attest to the accuracy of the approximate model. Numerical results for the model with consolidation are also presented and several insights are obtained. Section 6 concludes the paper and discusses possible directions for future research.

2. Model and assumption

Consider the single-item classical EOQ model with a deterministic and uniform demand having a rate of D units per year, a linear holding cost of h per unit per year, a fixed setup cost of K, and a unit production cost of c, where shortages are not allowed. Suppose that during the production of a lot of size y, the process goes out-of-control and starts producing imperfect quality items with a constant probability $q>0$ every time a unit of the item is produced. Once the process is out-of-control, it remains in this state until the entire lot is produced. This paper assumes that imperfect quality items are removed from the inventory at the end of a lot production. These items can be used in another production or retail system. For example, they can be sold at a lower price in the apparel industry.

The objective of the analysis in this section is to determine the optimal lot size, y^*, that minimizes the expected annual cost when imperfect quality items are withdrawn from the inventory. Deriving an expression for the expected annual cost requires determining the first two moments of the number of perfect quality items in a lot, X. Let $\bar{q}=1-q$. Lemma 1 gives the probability mass function and the first two moments of X.

Lemma 1. *The number of perfect quality (non-defective) items, X, in a lot of size y has the following pmf, first, and second moments*:

$$P\{X=j\}=\begin{cases} q\bar{q}^j, & j=1,2,\ldots,y-1 \\ \bar{q}^y, & j=y \end{cases}, \tag{1}$$

$$E[X]=\frac{\bar{q}(1-\bar{q}^y)}{q}, \tag{2}$$

$$E[X^2]=\frac{\bar{q}[(1+\bar{q})(1-\bar{q}^y)-2q\bar{q}^y y]}{q^2}. \tag{3}$$

Proof. See Appendix A.

Lemma 1 completely describes the distribution of the number of perfect quality items. This can be useful in other contexts involving, for example, inventory systems other than the EOQ setting.

Remark. As a validity check, and utilizing L'Hopital's rule, it can be shown that $E[X]$ and $E[X^2]$ are, respectively, equal to y and y^2 when $q\to 0$.