Computers and Mathematics with Applications 59 (2010) 2933–2939

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

Revisiting lot sizing for an inventory system with product recovery

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ARTICLE INFO

Article history:
Received 25 November 2009
Received in revised form 9 February 2010
Accepted 10 February 2010

Keywords:
Lot sizing
Rework
Algebraic approach

ABSTRACT

This note investigates the study by Teunter (2004) [1] on lot sizing for inventory systems with product recovery where lot sizing formulae for two recovery policies ($(1, R)$ and $(P, 1)$) are derived. Instead of applying the classical optimization technique, we develop an integrated solution procedure for each of the two policies using algebraic approaches. Numerical analysis show that our examples result in a lower total cost for both policies.

1. Introduction

Teunter [1] developed two lot sizing models with product recovery. In the first model, one production lot is alternated with R recovery lots, or $(1, R)$ policy. For the other model, P production lot is alternated with one recovery lot, or $(P, 1)$ policy. He derived two integrated total production inventory costs and three decision variables. They are the optimal production lot size (Q^*_p), the optimal recovery lot size (Q^*_r) and the number of recovery (R) or the production lots (P). The values of Q^*_p and Q^*_r are solved using partial differential equations. Corresponding to Q^*_p and Q^*_r, the values of R or P are then derived. Since R and P must be discrete, the author modified Q^*_p and Q^*_r, so that R or P was discrete. In this paper, we suggest an integrated solution procedure to solve Q^*_p and Q^*_r using a simple algebraic method without derivative. This method is simple and it is helpful for students who are not familiar with calculus.

There has been some research on solving an optimal solution without derivative and three methods are used widely. The methods are algebraic approach, cost-difference comparison method and arithmetic–geometric mean inequality. Grubbstorm [2] was the first to show that a standard economic order quantity model could be solved using an algebraic approach or without using derivative. Grubbstorm and Erdem [3] extended the approach by allowing backorder and Cardenas-Barron [4] applied the algebraic approach to solve the classical economic production quantity (EPQ) model with shortage. Yang and Wee [5] developed an integrated vendor–buyer inventory system derived without derivatives. Wee et al. [6] developed an EOQ model with temporary sale price derived without derivatives. Other researchers who used the algebraic approaches are Chang et al. [7] who solved EOQ and EPQ model with shortage, Sphicas [8] who solved EOQ and EPQ with linear and fixed backorder cost, Wee and Chung [9] who solved the economic lot size for an integrated vendor–buyer inventory system, Cardenas-Barron [10] who used the approach to solve an N-stage-multi-customer supply chain model and Cardenas-Barron [11] who solved inventory policies of immediate rework process model and N-cycle rework process model. Chung and Wee [12] developed an optimal economic lot size for a three-stage supply chain with backordering derived without derivatives. Cost-difference comparison method was introduced by Minner [13] and Wee et al. [14] extended the method by simplifying the solution procedure. Teng [15] was among the first researchers to derive

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0898-1221/$ – see front matter © 2010 Published by Elsevier Ltd
doi:10.1016/j.camwa.2010.02.011

Notations

d	demand rate (unit per time)
f	return fraction (unit per time)
p	production rate (unit per time)
r	recovery rate (unit per time)
K_p	ordering (setup) cost per production lot ($ per setup)
K_r	ordering (setup) cost per recovery lot ($ per setup)
h_r	holding cost per recoverable item per time unit ($ per unit per time)
h_s	holding cost per serviceable item per time unit ($ per unit per time)
Q_p	production lot size (unit)
Q_r	recovery lot size (unit)

Assumptions:

1. The return rate is equal to fd where $0 < f < 1$.
2. Production rate and recovery rate are larger than demand rate.
3. All return items are recovered.

EOQ using arithmetic–geometric mean inequality. Cardenas-Barron [16] extended the method and solved EOQ and EPQ model with backorder and Cardenas-Barron [17] presented a discussion on the use of arithmetic–geometric mean method.

2. Optimal solution without derivative

This section shows how the two models using algebraic approaches are solved. To compare our results with Teunter, we use the same notation and assumptions as [1].

2.1. Model 1: (1, R) policy

In the $(1, R)$ policy, manufacturer has one production setup and R rework setups per cycle. The following cost expression of the total inventory cost per unit time is obtained from Eq. (1) of [1]:

$$TC^{(1,R)}(Q_p, Q_r) = \frac{K_p d(1-f)}{Q_p} + \frac{K_r df}{Q_r} + \frac{h_s}{2}\left((1-f)\left(1-\frac{d}{p}\right)Q_p + f\left(1-\frac{d}{r}\right)Q_r\right) + \frac{h_r f}{2}\left(\left(1-\frac{d}{r}\right)Q_r + Q_p\right). \tag{1}$$

From Eq. (2) of [1],

$$RQ_r(1-f) = Q_p f. \tag{2}$$

After rearranging, one has:

$$Q_r = \frac{Q_p f}{R(1-f)}. \tag{3}$$

Teunter [1] differentiated (1) with respect to $Q^*{}_p$ and $Q^*{}_r$ and equating the result to zero, such that:

$$Q_p^{(1,R)} = \sqrt{\frac{2K_p d(1-f)}{h_s(1-f)\left(1-\frac{d}{p}\right)Q_p + h_r f}} \quad \text{and} \quad Q_r^{(1,R)} = \sqrt{\frac{2K_r d}{(h_s+h_r)\left(1-\frac{d}{r}\right)}}. \tag{4}$$

Since R has to be discrete, Teunter modify the optimal R so that the variable is discrete. The modification formulae for $(1, R)$ model is:

$$\tilde{Q_p}^{(1,R)} = \frac{\tilde{R}^{(1,R)} Q_r^{(1,R)}(1-f)}{f} \tag{5}$$

where

$$\tilde{R}^{(1,R)} = \max\{1, [R^{(1,R)}]\} \tag{6}$$

is the positive integer nearest to $R^{(1,R)}$.