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Lot-sizing and scheduling problem with earliness tardiness and setup penalties

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ABSTRACT

We consider the problem of determining the production lot sizes and their schedules so that the sum of setup cost, holding cost, and tardy cost is minimized. There are n orders waiting to be processed. Each order has its own due date, earliness penalty and tardiness penalty. The production is done in lots, which is the manufacturing of the same product continuously. No early delivery is allowed. Each order is delivered once, on the due date or immediately after the production of the order is completed. We present an algorithm, based on the use of the assignment problem, to optimally solve the problem with zero setup cost. For the problem with setup cost, the Backward Search algorithm is proposed to solve the small size problem. To deal with the large size problem, two heuristics which are Sequencing with Optimal Timing and Genetic Algorithm are presented.

1. Introduction

Production planning and scheduling is one of the most challenging tasks facing managers today. For companies involved in batch or lot production (for example, plastic injection, steel, or chemical production), planning production lot sizes for the finished products and deciding when to process them are two important problems requiring careful analysis in the production planning and scheduling. These can be termed the lot-sizing and scheduling problem. There are several lot-sizing and scheduling models being evolved for different situations. In traditional lot-sizing and scheduling models, the production lot sizes and their schedules should be made in such a way that demand is satisfied on time (no backorders) and the sum of total setup costs and total holding costs are minimized. Surveys of various lot-sizing and scheduling models can be found in the works of Bahl, Ritzman, and Gupta (1987); Drexl and Kimms (1997).

In the last decade, significant research involving scheduling problems under the Just-In-Time (JIT) philosophy has been conducted. With the JIT concept, customers are unwilling to receive early or tardy shipments. The most common scheduling objective is to minimize the deviation of job completion times around their due dates. A well known scheduling model involving the JIT concept is the earliness tardiness (E/T) scheduling problem. The objective is to determine the job schedule in order to minimize the sum of earliness and tardiness penalties. In the general E/T problem, the lot-sizing decision is not considered. A comprehensive review of E/T scheduling was provided by Baker and Scudder (1990).

The problem considered in this paper is to relax the assumption of no backorders and apply the concept of E/T problem to the lot-sizing and scheduling problem. This problem will be called the *earliness tardiness lot-sizing and scheduling* (ETLS) problem. Simply stated, the ETLS is the problem of determining the production lot sizes and deciding their schedules such that the sum of setup cost, holding cost and tardy cost is minimized. Similar to other lot-sizing and scheduling problems, the model assumes that delivery for each order (demand) is done only once. If the order is completed before or on its due date, it will be delivered on the due date. Otherwise, the order will be delivered immediately after its production is completed. The concept of discrete lot-sizing and scheduling problem (DLSP) along with the chemical batch scheduling will be applied to construct the ETLS model.

According to Hoesel and Kolen (1994), the DLSP problem which assumes a discrete time period, time varying demand, and finite planning horizon, is a suitable model for the short term or the operational level. The DLSP was first studied by Lasdon and Terjung (1971) with an application to production scheduling in a tire company. Complexity results for several types of DLSP were examined by Salomon, Kroon, Kuik, and Van Wassenhove (1991); Bruggeman and Jahnke (1997). Fleischmann (1990) presented a branch and bound procedure, using Lagrangean Relaxation for determining both lower bounds and feasible solutions, for solving the DLSP with sequence independent setup costs. Cattrysse, Salomon, Kuik, and Wassenhove (1993) extended the work of Fleischmann (1990) by including setup times. Both setup costs and setup times were sequence independent. In each period the machine is either in setup or in production for at most one item,

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or the machine is idle. The authors presented a heuristic based on dual ascent and column generation techniques for solving the problem. Fleishmann (1994) discussed the DLSP with sequence dependent setup costs. However, setup times were not included in the model. The authors presented a solution procedure based on the equivalence of the DLSP and a traveling salesman problem with time windows (TSPTW). Jordan and Drexl (1998) studied the DLSP with sequence dependent setup times and sequence dependent setup costs. They showed the equivalence between the DLSP and the batch sequencing problem (BSP). The authors proposed a branch and bound algorithm which is accelerated by bounding and dominance rules to solve the BSP model. Recently, the earliness and tardiness penalties in the batching and scheduling problem are discussed in batch chemical scheduling. Dessouky, Kijowski, and Verma (1999) studied the tradeoff between the earliness and tardiness penalties for a single-stage chemical processing environment with fixed batch sizes, sequence-independent setup times, and identical processing times. They presented a heuristic based on the transportation problem and assignment problem to solve the problem. The same problem but with sequence dependent setup costs was discussed by McGraw and Dessouky (2001).

2. Problem characteristics and notations

The following notations are used throughout the paper:

n	number of orders being placed at the beginning of planning horizon
o_i	demand (in unit of batches or production periods) of order i ($i = 1, \ldots, n$)
d_i	due date of order i ($i = 1, \ldots, n$)
D_i	delivery date of order i ($i = 1, \ldots, n$)
C_{ij}	completion time of batch j ($j = 1, \ldots, o_i$) of order i ($i = 1, \ldots, n$)
s_k	setup cost rate ($/setup) of product k ($k = 1, \ldots, p$)
α_i	holding cost rate ($/batch/period) of order i ($i = 1, \ldots, n$)
β_i	tardy cost rate ($/order/period) of order i ($i = 1, \ldots, n$)

The characteristics of ETLS are as follows:

- At the beginning of planning horizon, there are n orders waiting to be processed on a single machine.
- Each order is placed for one type of product. Different orders may be placed for either the same or different products. The demand for each order is in units of batches. Each *batch* is a number of units to be produced, at full machine capacity, within one production period (i.e., given that the production period represents a day, the demand of an order being placed for 20,000 U of product A, with the production rate of 1000 U/day, can be considered as a demand of 20 batches).
- Each order has its own due date (d_i), holding cost rate (α_i) in units of $/batch/period, and tardy cost rate (β_i) in units of $/order/period.
- To satisfy the demand of an order, production can be organized into production lots. A *production lot* is defined as a number of batches of the same product being processed continuously without idle periods. The demand of an order may be satisfied using more than one lot. On the other hand, it is possible that a lot may be used to satisfy more than one order. A setup is required before the production of a new lot of different products starts.
- Each order is delivered one time. If the production is completed before or on the due date, delivery will be on the due date. Otherwise, the order will be delivered immediately after its production is completed.

The problem assumptions can be stated as follows:

- Demands of each order are in the integer number of batches.
- For any order, $o_i \leqslant d_i$
- The holding cost of any batch will be considered after the batch is completely produced.
- For any period involving a delivery, the delivery occurs at the end of the period.

Example I shows the characteristics of the problem.

Example I. Assume that there are 3 orders waiting to be processed. The data is presented in Table 1.

By dividing the demand in units of product by the production rate, the demands of orders one, two, and three can be converted to 3, 2, and 5 batches, respectively. One production plan (which may not be optimal) is shown in Fig. 1.

The costs associated with the schedule in Fig. 1 can be calculated as follows:

Setup cost	there are three setups which occur at the beginning of lot #1, lot #3, and lot #4. Note that no setup is required at the beginning of lot #2 since it produces the same product as in lot #1. Therefore, the total setup cost incurred is \$3 + \$4 + \$3 = \$10
Tardy cost	only order three is tardy. Since it is tardy for 1 day, the total tardy cost is 1 × \$4 = \$4
Holding cost	for order one, there are 3 batches. The first batch is finished 2 days before the delivery date. The second batch is finished 1 day before the delivery date and the last batch is finished on the delivery date. Therefore, the holding cost of order one is $2\alpha_1 + 1\alpha_1 + 0$ = \$6. Using the same calculation, the holding cost of order two is $1\alpha_2 + 0$ = \$3 and the holding cost of order three is $6\alpha_3 + 5\alpha_3 + 2\alpha_3 + 1\alpha_3 + 0\alpha_3$ = \$28. Here, the total holding cost from this schedule is \$6 + \$3 + \$28 = \$37

The total cost incurred from this schedule is \$10 + \$4 + \$37 = \$51.

3. The ETLS with zero setup cost

This section is to discuss the problem with zero setup cost. Given that the decision variable C_{ij} ($i = 1, \ldots, n$; $j = 1, \ldots, o_i$) is the completion time of batch j of order i (the period in which batch j of order i is produced), the problem is to determine production lot sizes and their schedules in order to minimize the following objective function;

$$\min \sum_{i=1}^{n} \left(\sum_{j=1}^{o_i} [\alpha_i (D_i - C_{ij})] + [D_i - d_i] \right) \tag{1}$$

where $D_i = C_{io_i}$ if $C_{io_i} - d_i \geqslant 0$ and $D_i = d_i$, otherwise.

3.1. Problem properties

Property 1. If $d_i \neq D_i$ *and* $d_j \neq D_j$, then orders i and j cannot be delivered in the same period.

Table 1
The data for Example I.

Order	Product	Demand (U)	Production rate (U/day)	Due date	Setup cost ($/setup)	Holding cost ($/batch/day)	Tardy cost ($/batch/day)
1	A	3000	1000	5	3	2	2
2	B	10,000	5000	10	4	3	5
3	A	5000	1000	12	3	2	4