Expert Systems with Applications 37 (2010) 7234–7242

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Pricing and lot-sizing policies for deteriorating items with partial backlogging under inflation

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ARTICLE INFO

Keywords:
Pricing
Lot-sizing
Deteriorating items
Partial backlogging
Nelder–Mead algorithm

ABSTRACT

In this paper, we develop an inventory lot-size model for deteriorating items under inflation using a discounted cash flow (DCF) approach over a finite planning horizon. We allow not only a multivariate demand function of price and time but also partial backlogging. In addition, selling price is allowed for periodical upward and downward adjustments. The objective is to find the optimal lot size and periodic pricing strategies so that the net present value of total profit could be maximized. By using the properties derived from this paper and the Nelder–Mead algorithm, we provide a complete search procedure to find the optimal selling price, replenishment number and replenishment timing for the proposed model. At last, numerical examples are used to illustrate the algorithm.

1. Introduction

In the past 30 years, analysis of the inventory model allowing the constant demand rate to vary with time over a finite time horizon has extended the field of inventory control in practice. In the earlier period, researchers had discussed different demand patterns fitting the stage of product life cycle. Resh, Friedman, and Barbosa (1976), Donaldson (1977) considered the situation of linearly time-varying demand and established an algorithm to determine the optimal number of replenishments and timing. Barbosa and Friedman (1978, 1979) adjusted the lot-size problem for the cases of different increasing power-form demand functions and declining demand patterns, respectively. Henery (1979) further generalized the demand rate by considering a log-concave demand function increasing with time. Following the approach of Donaldson, Dave (1989) developed an exact replenishment policy for an inventory model with shortages. Yang, Teng, and Chern (2002) extended Barbosa and Friedman's (1978) model to allow for shortages. To characterize the more practical situation, Chen, Hung, and Weng (2007a, 2007b) dealt with the inventory model under the demand function following the product-life-cycle shape. They employed the Nelder–Mead algorithm to solve the mixed-integer nonlinear programming problem and determined the optimal number of replenishment and the optimal replenishment time points.

In real life, the deterioration phenomenon is observed on inventory items such as fruits, vegetables, pharmaceuticals, volatile liquids, and others. By considering this phenomenon occurring during the holding period, Dave and Patel (1981) integrated time-proportional demand and a constant rate of deterioration in the inventory model where shortages were prohibited over the finite planning horizon. Sachan (1984) further extended the model to allow for shortages. Later, Hariga (1996) generalized the demand pattern to any log-concave function. Researchers including Murdeshwar (1988), Goswami and Chaudhuri (1991), Goyal, Morin, and Nebebe (1992), Benkherouf (1995, 1998), Chakrabarti and Chaudhuri (1997) and Hariga and Al-Alyan (1997) developed economic order quantity models that focused on deteriorating items with time-varying demand and shortages.

However, the above inventory models unrealistically assume that during stockout period all demand is either backlogged or lost. In reality, some customers are willing to wait until replenishment, especially when the waiting period is short, while others are more impatient and go elsewhere. To reflect this phenomenon, Abad (1996) provided two sets of time-proportional backlogging rates: (i) linear time-proportional backlogging rate and (ii) exponential time-proportional backlogging rate. Chang and Dye (1999) developed a finite time horizon EOQ model in which the proportion of customers who would like to accept backlogging is the reciprocal of a linear function of the waiting time. Concurrently, Papachristos and Skouri (2000) established a partially backlogged inventory model by assuming that the backlogging rate decreases exponentially as the waiting time increases. Recently, Teng, Chang, Dye, and Hung (2002) and Chern, Yang, Teng, and Papachristos (2008) extended the fraction of unsatisfied demand backordered to any decreasing function of the waiting time up to the next replenishment.

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doi:10.1016/j.eswa.2010.04.004

Since price is viewed as an important vehicle to influence demand in most of the business environment, many researchers are led to investigate inventory models with a price-dependent demand. Cohen (1977) jointly determined the optimal replenishment cycle and price for deteriorating items with the demand rate dependent linearly on the selling price. Abad (1996, 2001, 2003, 2008) incorporated the demand rate described by any convex decreasing function of the selling price into the inventory model, taking a general rate of deterioration and a variable backlogging rate. Ho, Ouyang, and Su (2008) Chang, Ho, Ouyang, and Chia-Hsien Su (2009) presented the iso-elastic demand in an integrated supplier–buyer inventory model under the condition of a permissible delay in payment, respectively. The aforementioned studies assume firms have no pricing power to change the selling price periodically and adopt the fixed price policy. As opposed to the conventional fixed price policy, Chen and Chen (2004) presented an inventory model for deteriorating items with a multivariate demand function of price and time, taking account of the effects of inflation and time discounting over multiperiod planning horizon. However, the integer length of replenishment cycle is within certain limits due to the procedure using dynamic programming techniques.

It is noted that the literature about a finite time horizon inventory model rarely considers the cases with periodic adjustments of price. In this paper, we investigate the replenishment policies for deteriorating items with partial backlogging by considering a multivariate demand function of price and time. The fraction of unsatisfied demand backordered is any decreasing function of the waiting time up to the next replenishment. In addition, the selling price is allowed periodical upward and downward adjustments and the time value of money is taken into consideration. The objective of the inventory problem here is to determine the number of replenishments, the selling price per replenishment cycle, the timing of the reorder points and the shortage points. Following the properties derived from this paper, we provide a complete search procedure to find the optimal solutions by employing the Nelder–Mead algorithm. Several numerical examples are used to illustrate the features of the proposed model. At last, we make a summary and provide some suggestions for future research.

2. Assumptions and notation

2.1. Assumptions

The mathematical model of the inventory replenishment problem is based on the following assumptions:

1. The planning horizon of the inventory problem here is finite and is taken as H time units.
2. Lead time is zero.
3. The initial inventory level is zero.
4. A constant fraction of the on-hand inventory deteriorates per unit of time and there is no repair or replacement of the deteriorated inventory.
5. Shortages are allowed. The fraction of shortages backordered is a decreasing function $\beta(x)$, where x is the waiting time up to the next replenishment, and $0 \leqslant \beta(x) \leqslant 1$ with $\beta(0) = 1$. Note that if $\beta(x) = 1$ (or 0) for all x, then shortages are completely backlogged (or lost).

2.2. Notation

- θ the deterioration rate
- c_f the fixed purchasing cost per order
- p_i the selling price per unit (a decision variable) in the ith replenishment cycle, defined in the interval $[0,p_u]$, where p_u is the upper bound
- $f(t,p_i)$ the demand rate at time t and price p_i with $f(t,p_i) = g(t)A(p_i)$, where $g(t)$ is a positive and continuous function of time in the planning horizon $[0,H]$ and $A(p_i)$ is any non-negative, continuous, convex, decreasing function of the selling price in $[0,p_u]$
- c_v the purchasing cost per unit
- c_h the inventory holding cost per unit time
- c_s the backlogging cost per unit time due to shortages
- c_l the unit cost of lost sales
- n the number of replenishments over $[0,H]$ (a decision variable)
- t_i the ith replenishment time (a decision variable), $i = 1,2,\ldots,n$
- s_i the time at which the inventory level reaches zero in the ith replenishment cycle (a decision variable), $i = 1,2,\ldots,n$

As a result, the decision problem here has $3n$ decision variables.

3. Model formulation

According to the notations and assumptions mentioned above, the behavior of inventory system at any time can be depicted in Fig. 1. From Fig. 1, it can be seen that the depletion of the inventory occurs due to the combined effects of the demand and the deterioration during the interval $[t_i,s_i)$ of the ith replenishment cycle. Hence, the variation of inventory with respect to time can be described by the following differential equation:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -f(t,p_i) - \theta I(t), \quad t_i < t < s_i, \tag{1}$$

with boundary condition $I(s_i) = 0$, $i = 1,2,\ldots,n$. Solving the differential equation (1), we have

$$I(t) = e^{-\theta t}\int_t^{s_i} e^{\theta u} f(u,p_i)\,\mathrm{d}u, \quad t_i \leqslant t < s_i. \tag{2}$$

On the other hand, the depletion of inventory occurs due to the demand backlogged during the interval $[s_{i-1},t_i)$. The variation of inventory with respect to t can be described by the following differential equation:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -\beta(t_i - t)f(t,p_i), \quad s_{i-1} < t < t_i, \tag{3}$$

with boundary condition $I(s_{i-1}) = 0$, $i = 1,2,\ldots,n$. Solving the differential equation (3), we have

$$I(t) = -\int_{s_{i-1}}^{t} \beta(t_i - u)f(u,p_i)\,\mathrm{d}u, \quad s_{i-1} \leqslant t < t_i. \tag{4}$$

From (2), the present value of the holding cost in the ith cycle, denoted by HC_i, can be written as

$$HC_i = c_h \int_{t_i}^{s_i} e^{-rt} \int_t^{s_i} e^{\theta(u-t)} f(u,p_i)\,\mathrm{d}u\,\mathrm{d}t, \quad i = 1,2,\ldots,n. \tag{5}$$

The present value of the shortage cost due to shortages during $[(s_{i-1},t_i)]$ is

$$\begin{aligned} SC_i &= c_s \int_{s_{i-1}}^{t_i} e^{-rt} \int_{s_{i-1}}^{t} \beta(t_i - u) f(u,p_i)\,\mathrm{d}u\,\mathrm{d}t \\ &= \frac{c_s}{r}\int_{s_{i-1}}^{t_i} (e^{-rt} - e^{-rt_i})\beta(t_i - t)f(t,p_i)\,\mathrm{d}t, \quad i = 1,2,\ldots,n. \end{aligned} \tag{6}$$

The present value of the cost of lost sales during $[s_{i-1},t_i)$ is

$$LC_i = c_l \int_{s_{i-1}}^{t_i} e^{-rt}[1 - \beta(t_i - t)]f(t,p_i)\,\mathrm{d}t, \quad i = 1,2,\ldots,n. \tag{7}$$