Computers and Mathematics with Applications 60 (2010) 1895–1907

Contents lists available at ScienceDirect

Computers and Mathematics with Applications

journal homepage: www.elsevier.com/locate/camwa

A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and unit purchasing cost

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ARTICLE INFO

Article history:
Received 7 September 2009
Received in revised form 15 July 2010
Accepted 15 July 2010

Keywords:
Pricing
Inventory
Partial backlogging
Particle swarm optimization
Fluctuating cost
Inflation

ABSTRACT

In this paper, we extend the classical economic order quantity model to allow for not only a function of price-dependent and time-varying demand but also fluctuating unit purchasing cost. The joint replenishment problem is subject to continuous decay and a general partial backlogging rate. The objective is to find the optimal replenishment number, time scheduling and periodic selling price to maximize the discounted total profit. An effective search procedure is provided to find the optimal solution by employing the properties derived in this paper and particle swarm optimization algorithm. Several numerical examples are used to illustrate the features of the proposed model.

1. Introduction

In many inventory systems, the deterioration of goods is a realistic phenomenon. It is well known that certain products such as medicine, volatile liquids, blood, foodstuffs and many others, decrease under deterioration (vaporization, damage, spoilage, dryness and so on) during their normal storage period. As a result, while determining the optimal inventory policy of that type of products, the loss due to deterioration cannot be ignored. The fundamental result in the development of economic order quantity model with deteriorating items is from Ghare and Schrader [1] who established the classical no-shortage inventory model with a constant rate of decay. Based on Ghare and Schrader's [1] model, researchers including Wu et al. [2], Huang and Yao [3], Huang and Liao [4], Mishra and Mishra [5], Maity and Maiti [6], Geetha and Uthayakumar [7] and Chang et al. [8] developed economic order quantity models that focused on deteriorating items.

Furthermore, in classical inventory models, the economic order quantity (EOQ) formula is assumed to be a constant demand rate and a fixed unit purchasing cost, and therefore offers ease of use and applications. In reality, the demand may vary with product life cycle duration. The assumption of a constant demand rate is usually valid in the maturity stage. In literature, the demand rate had been well approximated by specific forms to indicate the stage of a product in its life cycle. As Goyal and Giri [9] pointed out, most of the time-varying demand inventory models considered either linearly increasing/decreasing demand (i.e., $f(t) = a + bt$, with $a > 0, b \neq 0$) or exponentially increasing/decreasing demand (i.e., $f(t) = ae^{bt}$, with $a > 0, b \neq 0$) patterns. We refer the reader to their references therein for more details. Recently, Chen et al. [10], Chen et al. [11,12] dealt with the inventory model under the demand function following the product-life-cycle shape over a fixed time horizon.

Moreover, the assumption of a fixed unit purchasing cost does not reflect the situation where the inflation rate is high or the situation where price increase or decrease is expected. With the advances in technology and global division of labor, the

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doi:10.1016/j.camwa.2010.07.023

unit cost of high-tech products might drop due to the introduction of new products. In the personal computer (PC) industry, Lee et al. [13] showed components' price declines constantly over the product life cycle. Under an exponential cost decrease but a constant demand rate, Khouja and Park [14] analyzed the problem of optimizing the lot size with equal length for the entire horizon. Teunter [15] then developed a net present value formulation of Khouja and Park's [14] model, and derived a simple modified EOQ formula. Khouja and Goyal [16] relaxed the restriction of equal length to allow varying cycle times.

Besides the continuous decrease of purchasing cost, in real-life situations, the gasoline price or raw material price may be going up constantly. When the cost of purchases as a percentage of sales is often substantial, it is necessary to include a fluctuating purchasing cost for the inventory system. Khouja et al. [17] developed the joint replenishment problem to analyse the effect of continuous unit purchasing cost decrease or increase on the optimal ordering frequencies. In contrast to the traditional EOQ model, Teng and Yang [18] assumed that both the demand function and the unit purchasing cost are fluctuating with time, which are more general than increasing, decreasing, and log-concave functions. Teng et al. [19] then provided an easy-to-use algorithm to find the optimal replenishment number and schedule for complete shortages. Teng and Yang [20] further allowed for time-varying purchasing cost and generalized holding cost over a finite-planning horizon.

From the competitive standpoint of the business, maximizing profit plays an important role for getting and keeping a successful position in a competitive market. However, the above inventory models subject to decreasing or increasing unit purchasing cost are developed to minimize the total relevant cost. To achieve profit maximization, Chen and Chen [21] presented an inventory model for a deteriorating item with a multivariate demand function of price and time but a fixed unit purchasing cost. Their model is solved by dynamic programming techniques to adjust the selling price upward or downward periodically. Chang et al. [22] established an inventory model for a retailer to determine its optimal selling price, replenishment number and replenishment schedule. They also assume a fixed unit purchasing cost, but the existence and uniqueness of the maximum solution is obtained under the same selling price per cycle. Similarly, when the unit purchasing cost is fluctuating with time, a decision maker needs to adjust its pricing strategy, but the joint pricing and replenishment policy is seldomly discussed.

In this paper, we assume that unit purchasing cost is positive and fluctuating with time. We investigate the replenishment policies for a deteriorating item with partial backlogging by considering a multivariate demand function of price and time and the effect of discount rate over multiperiod planning horizon. The fraction of unsatisfied demand backordered is any decreasing function of the waiting time up to the next replenishment. In addition, the selling price is allowed for periodical upward and downward adjustments. The objective of the inventory problem here is to determine the number of replenishments, the selling price per replenishment cycle, the timing of the reorder points and the shortage points. Following the properties derived from this paper, we provide a complete search procedure to find the optimal solutions by employing the search method based on particle swarm optimization algorithm. Several numerical examples are used to illustrate the features of the proposed model. At last, we make a summary and provide some suggestions for future research.

2. Assumptions and notation

The mathematical model in this paper is developed on the basis of the following assumptions and notations:

2.1. Assumptions

1. A single item is considered with a constant rate of deterioration over a known and finite planning horizon of length H.
2. The replenishment occurs instantaneously at an infinite rate.
3. There is no repair or replacement of deteriorated units during the planning horizon. The items will be withdrawn from the warehouse immediately as they deteriorate.
4. Shortages are allowed in all cycles and each cycle starts with shortages.
5. The fraction of shortages backordered is a decreasing function $\beta(x)$, where x is the waiting time up to the next replenishment, and $0 \leq \beta(x) \leq 1$ with $\beta(0) = 1$. Note that if $\beta(x) = 1$ (or 0) for all x, then shortages are completely backlogged (or lost).

2.2. Notation

$n =$ The number of replenishment cycles during the planning horizon (a decision variable)
$\theta =$ the deterioration rate
$r =$ the discount rate
$c_f =$ the ordering cost per order
$c_v(t) =$ the unit purchasing cost at time t, where $c_v(t)$ is a positive and continuous function of time in the planning horizon
$p_i =$ the selling price per unit (a decision variable) in the ith replenishment cycle, defined in the interval $[0, p_u]$
$f(t, p_i) =$ the demand rate at time t and price p_i with $f(t, p_i) = g(t)A(p_i)$, where $g(t)$ is a positive and continuous function of time in the planning horizon and $A(p_i)$ is any non-negative, continuous, convex, decreasing function of the selling price in $[0, p_u]$