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A particle swarm optimization for solving joint pricing and lot-sizing problem with fluctuating demand and trade credit financing

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ABSTRACT

Pricing is a major strategy for a retailer to obtain its maximum profit. Furthermore, under most market behaviors, one can easily find that a vendor provides a credit period (for example 30 days) for buyers to stimulate the demand, boost market share or decrease inventories of certain items. Therefore, in this paper, we establish a deterministic economic order quantity model for a retailer to determine its optimal selling price, replenishment number and replenishment schedule with fluctuating demand under two levels of trade credit policy. A particle swarm optimization is coded and used to solve the mixed-integer nonlinear programming problem by employing the properties derived in this paper. Some numerical examples are used to illustrate the features of the proposed model.

1. Introduction

In the inventory models developed, it is often assumed that payment will be made to the vendor for the goods immediately after receiving the consignment. Because the permissible delay in payments can provide economic sense for vendors, it is possible for a vendor to allow a certain credit period for buyers to stimulate the demand to maximize the vendors-owned benefits and advantage. Recently, several researchers have developed analytical inventory models with consideration of permissible delay in payments. Goyal (1985) first studied an EOQ model under the conditions of permissible delay in payments. Chung (1989) presented the discounted cash flows (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. Later, Shinn, Hwang, and Sung (1996) extended Goyal's (1985) model and considered quantity discounts for freight cost. Chung (1997) presented a simple procedure to determine the optimal replenishment cycle to simplify the solution procedure described in Goyal (1985). Teng (2002) provided an alternative conclusion from Goyal (1985), and mathematically proved that it makes economic sense for a well-established buyer to order less quantity and take the benefits of the permissible delay more frequently. Huang (2003) developed an EOQ model in which a supplier offers a retailer the permissible delay period M, and the retailer in turn provides the trade credit period N (with $N \leqslant M$) to his/her customers. He then obtained the closed-form optimal solution for the problem.

Jaber and Osman (2006) proposed a two-level supply chain model with delay in payments to coordinate the players' orders and minimize the supply chain costs. Jaber (2007) then incorporated the concept of entropy cost into the EOQ problem with permissible delay in payments. In real situations, "time" is a significant key concept and plays an important role in inventory models. Certain types of commodities deteriorate in the course of time and hence are unstable. As a result, while determining the optimal inventory policy for product of that type, the loss due to deterioration cannot be ignored. To accommodate more practical features of the real inventory systems, Aggarwal and Jaggi (1995) and Hwang and Shinn (1997) extended Goyal's (1985) model to consider the deterministic inventory model with a constant deterioration rate. Since the occurrence of shortages in inventory is a very nature phenomenon in real situations, Jamal, Sarker, and Wang (1997), Sarker, Jamal, and Wang (2000), Chang and Dye (2000), Chang, Hung, and Dye (2002) extended Aggarwal and Jaggi's (1995) model to allow for shortages and makes it more applicable in real world. Chang, Ouyang, and Teng (2003) then extended Teng's (2002) model, and established an EOQ model for deteriorating items in which the supplier provides a permissible delay to the purchaser if the order quantity is greater than or equal to a predetermined quantity. By considering the difference between unit selling price and unit purchasing cost, Ouyang, Chuang, and Chuang (2004) developed an EOQ model with noninstantaneous receipt under conditions of permissible delay in payments. Recently, Taso and Sheen (2007) developed a finite time horizon inventory model for deteriorating items to determine the most suitable retail price and appropriate replenishment cycle time with fluctuating unit purchasing cost

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and trade credit. Chang, Wu, and Chen (2009) established an inventory model to determine the optimal payment time, replenishment cycle and order quantity under inflation.

However, all the above models make an implicit assumption that the demand rate is constant over an infinite planning horizon. This assumption is only valid during the maturity phase of a product life cycle. During the introduction and growth phase of a product life cycle, the firms face increasing demand with little competition. Some researchers Resh, Friedman, and Barbosa (1976), Donaldson (1977), Dave and Patel (1981), Sachan (1984), Goswami and Chaudhuri (1991), Goyal, Morin, and Nebebe (1992), Chakrabarty, Giri, and Chaudhuri (1998) suggest that the demand rate can be well approximated by a linear form. A linear trend demand implies an uniform change in the demand rate of the product per unit time. This is a fairly unrealistic phenomenon and it seldom occurs in the real market. One can usually observe in the electronic market that the sales of items increase rapidly during the introduction and growth phase of the life cycle because there are few competitors in market. Recently, Yang, Teng, and Chern (2002) established an optimal replenishment policy for power-form demand rate and proposed a simple and computationally efficient method in a forward recursive manner to find the optimal replenishment strategy. Khanra and Chaudhuri (2003) advise that the demand rate should be represented by a continuous quadratic function of time in the growth stage of a product life cycle. They also provide a heuristic algorithm to solve the problem when the planning horizon is finite. To achieve maximum profit, Chen and Chen (2004) presented an inventory model for a deteriorating item with a multivariate demand function of price and time. Their model is solved with dynamic programming techniques by adjusting the selling price upward or downward periodically. Chen, Hung, and Weng (2007a, 2007b) dealt with the inventory model under the demand function following the product-life-cycle shape over a fixed time horizon. Skouri and Konstantaras (2009) studied an order level inventory model when the demand is described by a three successive time periods that classified time dependent ramp-type function.

In this paper, to obtain robust and general results, we will extend the constant demand to a generalized time varying demand, which is suitable not only for the growth stage but also for the maturity stage of a product life cycle. In addition, we assume that supplier offers retailer a trade credit period M, and retailer in turn provides a trade credit period N (with $N \leqslant M$) to his/her customers. The lot sizing problem is then to find the optimal pricing and replenishment strategy that will maximize the present value of total profit. A traditional particle swarm optimization is coded and used to solve the mixed-integer nonlinear programming problem by employing the properties derived in this paper. Finally, numerical examples will be used to illustrate the results.

2. Assumptions and notations

The mathematical model in this paper is developed on the basis of the following assumptions and notations.

2.1. Notations

$I(t)$ = the inventory level at time t.
A = ordering cost, \$/per order.
c = unit purchasing cost, \$/per unit.
p = unit selling price (a decision variable), \$/per unit, defined in the interval $[0,p_u]$.
$g(t,p)$ = the demand rate at time t and price p with $g(t,p) = \alpha(p)f(t)$, where $f(t)$ is positive in the planning horizon $[0,H]$ and $\alpha(p)$ is a non-negative, continuous, convex, decreasing function of the selling price in $[0,p_u]$.
r = the discount rate.
h = holding cost excluding interest charges, \$/unit/year.
I_e = interest which can be earned, \$/year.
I_r = interest charges which are invested in inventory, \$/year.
M = the retailer's trade credit period offered by supplier in years.
N = the customer's trade credit period offered by retailer in years, where $N \leqslant M$.
n = the number of replenishment periods during the planning horizon.
t_i = the ith replenishment time (a decision variable), $i = 1,2,\ldots,n$, with $0 = t_0 < t_1 < t_2 < \cdots < t_n = H$.
T_i = the length of ith replenishment period.
Q_i = the order quantity in the ith replenishment period.
$TP(n,p,\mathbf{t})$ = the present value of total profit, where $\mathbf{t} = \{t_1,t_2,\ldots,t_{n-1}\}$.

2.2. Assumptions

1. The inventory system involves in only one item over a known and finite planning horizon H.
2. The replenishment occurs instantaneously at an infinite rate.
3. The items deteriorate at a constant rate of deterioration θ, where $0 < \theta \ll 1$. There is no repair or replacement of deteriorated units during the planning horizon. The items will be withdrawn from the warehouse immediately as they deteriorate.
4. Before the replenishment account is settled, the retailer can use the sales revenue to earn interest with an annual rate I_e. However, beyond the fixed credit period, the product still in stock is presumed to be financed with an annual rate I_r.
5. The retailer can accumulate revenue and earn interest after his/her customer pays for the amount of purchasing cost to the retailer until the end of the trade credit period offered by the supplier. That is, the retailer can accumulate revenue and earn interest during the period N to M with rate I_e under the condition of trade credit.

3. Model formulation

As shown in Fig. 1, the depletion of the inventory occurs due to the combined effects of the demand and deterioration in the interval $[t_{i-1},t_i]$. Hence, the variation of inventory level, $I(t)$, with respect to time can be described by the following differential equation:

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -\theta I(t) - \alpha(p)f(t), \quad t_{i-1} \leqslant t < t_i, \tag{1}$$

with boundary condition $I(t_i) = 0$, $i = 1,2,\ldots,n$. The solution of (1) can be represented by

$$I(t) = e^{-\theta t}\int_t^{t_i} \alpha(p)f(t)e^{\theta u}\,\mathrm{d}u, \quad t_{i-1} \leqslant t < t_i. \tag{2}$$

Then, applying (2), the present value of the holding cost in the ith replenishment period, denoted by HC_i, $i = 1,2,\ldots,n$, can be written as

$$HC_i = h\int_{t_{i-1}}^{t_i} e^{-rt}e^{-\theta t}\int_t^{t_i} \alpha(p)f(t)e^{\theta u}\,\mathrm{d}u\,\mathrm{d}t. \tag{3}$$

The present value of the purchase cost during the ith replenishment period, denoted by PC_i, $i = 1,2,\ldots,n$, is

$$PC_i = ce^{-rt_{i-1}}\int_{t_{i-1}}^{t_i} \alpha(p)f(t)e^{\theta(t-t_{i-1})}\,\mathrm{d}t. \tag{4}$$