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A variable neighborhood search based approach for uncapacitated multilevel lot-sizing problems

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ABSTRACT

In this paper, an effective approach based on the variable neighborhood search (VNS) algorithm is presented to solve the uncapacitated multilevel lot-sizing (MLLS) problems with component commonality and multiple end-items. A neighborhood structure for the MLLS problem is defined, and two kinds of solution move policies, i.e., move at first improvement (MAFI) and move at best improvement (MABI), are used in the algorithm. A new rule called *Setup shifting* is developed to conduct a more efficient neighborhood search for the MLLS problems. Computational studies are carried out on two sets of benchmark problems. The experimental results show that the VNS algorithm is capable of solving MLLS problems with good optimality and high computational efficiency as well, outperforming most of the existing algorithms in comparison.

1. Introduction

The multilevel lot-sizing (MLLS) problem concerns how to determine the lot sizes for producing/procuring multiple items at different levels, which are quantitative interdependent, so as to minimize the production/procurement setup cost and inventory-holding cost in the whole planning horizon. High quality solution of the MLLS problem can improve efficiently the operation of modern manufacturing and assembly processes. Algorithms providing optimal solutions exist for the problem; however, only small instances can be solved in reasonable time because the problem is NP-hard (Steinberg & Napier, 1980). Several optimization formulations and algorithms have been developed to solve variant MLLS problems. Early dynamic programming formulations used a network representation of the problem with a series structure (Zhangwill, 1968, 1969) or an assembly structure (Crowston & Wagner, 1973). Other approaches involve the branch and bound algorithms (Afentakis, Gavish, & Kamarkar, 1984, 1986) that used a converting approach to change the classical formulation of the general structure into a simple but expanded assembly structure. As the MLLS problem is so common in practice and the solution plays a fundamental role in MRP system, many heuristic approaches have also been developed, consisting first of the sequential application of the single-level lot-sizing models to each component of the product structure (Veral & LaForge, 1985; Yelle, 1979), and later, of the application of the multilevel lot-sizing models. The multilevel models quantify items interdependencies and thus perform better than the single-level based models (Blackburn & Millen, 1982, 1985; Coleman & McKnew, 1991).

In recent years, the so-called meta-heuristic algorithms have been developed to solve the MLLS problems, such as the hybrid genetic algorithm (Dellaert & Jeunet, 2000; Dellaert, Jeunet, & Jonard, 2000), the simulated annealing (Raza & Akgunduz, 2008; Tang, 2004), the particle swarm optimization (Han, Tang, Kaku, & Mu, 2009), the soft optimization approach based on segmentation (Kaku, Li, & Xu, 2010; Kaku & Xu, 2006), and the ant colony optimization system (Almeder, 2010; Pitakaso, Almeder, Doerner, & Hartlb, 2007). It have been reported that these algorithms can provide highly cost-efficient solutions within reasonable time. However, most of the meta-heuristic algorithms, such as the hybrid genetic algorithm (HGA), the particle swarm optimization (PSO) and the ant colony optimization (ACO), are highly technology-based and require complicated programming skills, which as a consequence might deter many potential users, not mentioning the mathematical complexity of these algorithms. The soft optimization (SO) approach may be easy for implementation, but is not very effective in finding solutions with good qualities.

In this paper, we present a succinct approach based on the variable neighborhood search (VNS) algorithm to efficiently solve the uncapacitated MLLS problem. The VNS algorithm, initiated by

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Mladenovic and Hansen (1997), is a top-level methodology for solving the combinatorial optimization problems. Since its principle, as to be shown in Section 3 is very simple and easily understood and implemented, the VNS algorithm has been successfully applied to solve various kinds of optimization problems, such as Travel Salesman Problem (Mladenovic & Hansen, 1997), P-media problem (Fathali & Kakhki, 2006; Hansen & Mladenovic, 1997), and Vehicle Routing Problem (Chen, Huang, & Dong, 2010; Imran, Salhi, & Wassan, 2009). The success of VNS is largely due to its enjoying most of the 11 desirable properties of meta-heuristics generalized by Hansen, Mladenovic, and Pérez (2008), such as simplicity, user-friendliness, efficiency, effectiveness, etc. Since the MLLS problem is observed to share common characteristics, e.g., binary decision variables, with the problems solved successfully by VNS based algorithms, it is promising to develop a VNS based algorithm for efficiently solving the MLLS problem. To our best knowledge, this work is a first attempt to solve the classical MLLS problem by using a VNS based algorithm and fortunately, the VNS algorithm does not disappoint us, showing its broad applicability again on combinatorial optimization problem.

In practice, various models on the MLLS problems have been developed to match the practical production environments, such as the capacitated lot-sizing model with variable technology by Pratsini (2000). In this paper, we focus on the basic model of the MLLS problem that is featured as uncapacitated, with time-invariant costs, of finite horizon, with no backlogging, and with static requirements on end-products. To examine the performance of our new algorithm, different kinds of product structures are considered in the experimental studies. The experiments are conducted on two sets of benchmark problems — 96 small-size problems and 40 medium-size problems, to compare the new algorithm with the existing hybrid genetic algorithm (HGA) (Dellaert & Jeunet, 2000), the MAX–MIN ant system (MMAS) algorithm (Pitakaso et al., 2007), and the parallel GA algorithm (PGA) (Homberger, 2008). The results show that the VNS algorithm is very competitive since it can on average find better solutions in less computing time.

The rest of the paper is organized as follows. Section 2 describes the MLLS problem. Section 3 explains the principle of the VNS algorithm, the definition of the neighborhood structure for the MLLS problem, a rule named *setup shifting* for considering interdependencies, and the scheme of the new VNS algorithm for MLLS problem. In Section 4, computational experiments are carried out to test the new algorithm against existing algorithms. Finally, in Section 5, we conclude the paper.

2. The MLLS problem

The MLLS problem under investigation is considered to be uncapacitated, discrete-time, occurring in a multilevel production/inventory system with general product structure. We assume that external demands for the end-items are known up to the planning horizon, and backlog is not allowed. Suppose that there are m items and the planning horizon is divided into n periods. The purpose is to find the optimal production setups and the lot sizes of all items for the minimization of total setup cost and inventory-holding cost over the n-period planning horizon, while ensuring that all external demands must be met.

The MLLS problem can be formulated as an integer optimization model. We use the notations in Dellaert and Jeunet (2000) to formulate it as follows.

i: Index of item, $i = 1, 2, \ldots, m$,
t: Index of period, $t = 1, 2, \ldots, n$,
h_i: Unit inventory-holding cost per period for item$i$,
s_i: Setup cost for item i,
Γ_i: The set of immediate successors of item i,
Γ_i^{-1}: The set of immediate predecessors of item i,
D_{it}: Requirements for item i in period t,
I_{it}: Inventory level of item i at the end of period t,
l_i: Leading time to assemble, to manufacture or to purchase item i,
p_{it}: Production quantity for item i in period t,
M: A large number.
x_{it}: Binary decision index representing the setup of item i in period t, i.e., $x_{it} = 1$ if item i is setup in period t and $x_{it} = 0$ otherwise.

The objective function is the sum of setup cost and inventory-holding costs for all items over the planning horizon, denoted by TC (total cost), which is formulated as follows.

$$TC = \sum_{i=1}^{m}\sum_{t=1}^{n}(h_i \cdot I_{i,t} + s_i \cdot x_{i,t}). \tag{1}$$

The MLLS problem is to minimize TC subject to the following constraints.

$$I_{it} = I_{i,t-1} + p_{i,t} - D_{i,t}, \tag{2}$$

$$D_{i,t} = \sum_{j \in \Gamma_i} C_{i,j} \cdot p_{j,t+l_j} \quad \forall i | \Gamma_i \neq \phi, \tag{3}$$

$$p_{i,t} - M \cdot x_{i,t} \leqslant 0, \tag{4}$$

$$I_{i,t} \geqslant 0, \quad p_{i,t} \geqslant 0, \quad x_{i,t} \in \{0,1\}, \quad \forall i,t \tag{5}$$

In the above constraints, Eq. (2) expresses the inventory flow conservation constraint for item i. Note that if item i is an end product (characterized by $\Gamma_i = \varphi$), its demand is exogenously given, whereas if it is a component part ($\Gamma_i \neq \varphi$), its demand is defined by the production of its successors (items belonging to Γ_i). Eq. (3) guarantees that the demand for item i in period t results from the exact sum of lot sizes of its successors (items belonging to Γ_i) multiplied by production ratio with leading time correction. Constraint (4) guarantees that a setup cost is incurred when a production is arranged. Constraint (5) states that backlog is not allowed, and production is either positive or zero, and the setup decision variables are binary.

Basically, in the optimal solution of a MLLS problem, there exists

$$x_{i,t} \cdot I_{i,t-1} = 0, \tag{6}$$

which indicates that any optimal lot size must cover the total demand of an integer number of periods and the production setup happens only when the inventory level drops to zero. Commonly, for all items, since initial inventory levels are zero (indicated by $I_{i,0} = 0$), their first periods with positive demand must keep being setup for production/purchase, to ensure that the solution is always feasible.

For pure assembly structure,[1] the *inner corner* property had been observed in the optimal solution of MLLS problem with zero leading time (Tang, 2004), which was reported to enhance the solution efficiency by using the following constraint.

$$x_{i,t} \geqslant x_{k,t}, \quad \forall k \in \Gamma_i^{-1}. \tag{7}$$

The difficulty of solving MLLS problem has been recognized for decades and is still a challenge of today because the size of its solution space will exponentially accelerate to be considerably huge as the problem's size increases. What is more, nowadays the MLLS problems are continuously increasing in both product complexity and problem size. Although the feasible solution space may be

[1] In pure assembly structure, each item has multiple immediate predecessors but has at most only one direct successor; in general structure, each item may have multiple immediate predecessors and multiple direct successors.