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Non-cooperative strategies for production and shipment lot sizing in one vendor–multi-buyer system

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A R T I C L E I N F O

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A B S T R A C T

The supply chain structure examined in this paper consists of a single vendor (or manufacturer) with multiple heterogeneous buyers (or retailers). A continuous deterministic model is presented. To satisfy buyers demands, the vendor will deliver the product in JIT shipments to each buyer. The production rate is constant and sufficient to meet the buyers' demands. The product is shipped in discrete batches from the vendor's stock to buyers' stocks and all shipments are realized instantaneously. Special production–replenishment policies of the vendor and the buyers are analyzed. That is, the production batch is transferred to each buyer in several sub-batches in each production distribution cycle (PDC).

This paper offers game model without prices, where agents minimize individual costs. It is a non-cooperative $(1+N)$-person game model with agents (a single vendor and N-buyers) choosing numbers and sizes of transferred batches. The model describes inventory patterns and cost structure of PDC. It is proved that there exist Nash equilibria in several types of sub-games of the considered game.

1. Introduction

One of the major tasks in supply chain management is to coordinate the processes in the supply chain to obtain lower system-wide cost. In general, a supply chain is composed of independent partners with individual costs. For this reason, each firm (partner) is interested in minimizing its own cost independently. Both, in practice and in the literature considerable attention is paid to the coordination of flows between distinct entities (as supplier, manufacturer, transporter, buyer, etc.) in supply chain.

The idea of joint optimization for vendor and buyer was initiated by Goyal (1976) and Banerjee (1986). A basic policy is any feasible policy where deliveries are made only when the buyer has zero inventory. Several authors incorporated policies in which sizes of successive shipments from the vendor to the buyer within a production cycle either increases by a factor (equal to the ratio of production rate to the demand rate) or are equal in size. For one buyer case, Hill (1999) shows that in the optimal PDC, the production batch first is transferred in increasing size and then in equals size of sub-batches.

In most papers dealing with integrated inventory models, the transportation cost is considered only as a part of fixed setup or replenishment cost. Ertogral et al. (2007) have studied how the results of incorporation of transportation cost into the model influence on better decision making under equal size shipment policies. A fundamental advance in the two-side cost structure is in recognizing how delivery-transportation costs apply to both sides. David and Eben-Chaime (2003) and Kelle et al. (2003) suggested such a separation for policies with respect to equal in size deliveries. Some ideas for partition of delivery-transportation costs in more general case was given in Bylka (2003). However, there is an additional set of problems involved in implementing policies (strategies) with respect to whether and how the agents participate in the delivery-transportation costs in multiple buyers case. For the case with deliveries of equal sizes, some solutions can be find in a number of papers, including Banerjee et al. (2007), Chan and Kingsman (2007) and Tang et al. (2008). This paper presents a solution in the case with non-equal size deliveries. Other related papers have been developed by Siajadi et al. (2006) and Abdul-Jalbar et al. (2007). A comprehensive literature review of related works in this field is presented in Sarmah et al. (2006). The general result of this type of papers is that cooperations reduce the total system cost.

It is not a typical case that suppliers and buyers coordinate their production and ordering–shipment policies. Some researchers, for example Kelle et al. (2003) have presented quantitative results which can serve as a motivation and negotiating tool for suppliers and buyers to coordinate their decisions. It is natural that potential savings in cooperation (in centralized case) cannot be ignored. Competitive pressures drive profits down. Perhaps, it forces firms to reduce costs while maintaining excellent customer service. Most studies on game theory models of supply chain consider agents which maximize individual profit functions (with respect to purchase and sale prices). Bylka (2003, 2009)

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investigated equilibrium strategies in non-cooperative game under the assumption, that only the division of the transportation cost with respect to the numbers of shipments is centrally coordinated or negotiated before the game.

The research presented in this paper offers a game model without prices, where agents minimize individual costs. It is a non-cooperative game model with agents (a single vendor and N-buyers) choosing numbers and sizes of transferred batches. It is a generalization of the paper Bylka (2009). The remainder of the paper is organized as follows. In Section 2, the model describing inventory patterns and cost structure under a production distribution cycle (PDC) is developed. Then, it is assumed that the players (the vendor and the buyers) choose decisions through a given sub-game of considered game. The existence of Nash equilibrium strategies is proved in Sections 3 and 4.

2. Modelling vendor–buyer relationships under a production distribution cycle (PDC)

We consider a continuous deterministic model of a production–distribution system for a single product. In the model notation, the vendor's and buyers' parameters are indexed with $i=0$ for the vendor and $i=1,\ldots,N$ for buyers, respectively. Buyers' demands are continuous functions of the time. The vendor produces a product and supplies it to the buyers in discrete batches (as in Goyal, 1995; Banerjee and Burton, 1994).

2.1. Model description and assumptions

The problem will be characterized by the following assumptions:

A1. The production rate $P>0$ is constant and sufficient to meet the total buyers' demand with the rate $D=D_1+\cdots+D_N$, where $D_1>0,\ldots,D_N>0$ are individual demand rates. In other words $\lambda=P/D>1$.

A2. The final product is distributed by shipping it in discrete lots from the vendor's stock to buyers' stocks (realized instantaneously).

A3. The fixed production set up cost is denoted by A and the fixed ordering/shipment cost for i-th agent will be denoted by A_i. The stock holding costs are linear with unit cost h_i, for $i=0,1,\ldots,N$. Additionally, $h_0<h_i$ for each $i>0$.

A4. There is exactly one production set up (for production batch $Q>0$) at the beginning of the production–distribution cycle (PDC) with the length $T=Q/D$ and production time $t^*=Q/P$. Additionally, for each i-th agent the inventory position $I_i(t)$ is nonnegative for any time moment $t\in[0,T]$. The initial inventory positions are the same as the final ones, i.e. $I_i(0)=I_i(T)$.

In the case of central coordination, the problem is to find a schedule which minimizes the average total (production, shipping, replenishment and holding inventory) common cost for a given (or infinite) time horizon. For large time horizon, the production–distribution schedule is a sequence of PDC. For infinite time horizon, an optimal schedule contains only cycles with minimal average costs—called economic production–distribution cycles (EPDC).

In this paper we investigate a case without central coordination. Additionally we assume:

A5. Each buyer receives shipments just in time to run out of the stock (replenishment only if inventory positions stay zero). For each buyer i the last $k_i\geq 0$ shipments have the same sizes.

A6. After each of the $k_0\geq 0$ initial shipments the vendor's stock becomes empty.

A7. The vendor controls k_0 initial shipments (with respect to its own cost of the transported lots) and each buyer i controls own k_i last shipments.

The assumptions A1–A4 are reasonable, they are based on the inspirations found in the practice as well as in the most integrated inventory models in the literature (see presentation in Sarmah et al., 2006). One consequence of them is that in the centralized case, the EPDC satisfies assumptions A5–A7 for one buyer model. For convenience, in this paper we assume that A5–A7 hold.

For one buyer case, Hill (1999) derived the structure of the EPDC in the centralized (cooperative) case. It satisfies the following: The production batch is distributed so that *initial increasing (proportionally to λ) in sizes k_0 sub-batches precede k_1 sub-batches equal in sizes*. These policies were investigated as non-cooperative agents' strategies in Bylka (2003), where the structure of equilibrium economic production–distribution cycle (EEPDC) was presented.

To precise the assumptions A1–A7 we use standard mathematical notation with $\mathcal{R}_+$ and $\mathcal{N}$ as nonnegative real and natural numbers, respectively. Additionally,

- $\xi=(P,D_1,\ldots,D_N)\in\mathcal{R}_+^{N+1}$ and $\eta=(A,A_0,A_1,\ldots,A_N,h_0,h_1,\ldots,h_N)$ will be denoted technological (see A1) and cost (see A3) parameters of the model, respectively;
- $Q\in\mathcal{R}_+$, $\tilde{M}=(M_1,\ldots,M_N)\in\mathcal{R}_+^N$ and $\tilde{k}=(k_0,k_1,\ldots,k_N)\in\mathcal{N}_+^{N+1}$ are decision parameters;
- $I_i(t)$ describe the i-th agent's inventory positions at the time moment $t\in[0,T]$ before the possible replenishment; $I_i^+(t)$ describe the inventory positions just past replenishment, if any;
- $q_{i,0}=I_i(0)$ denotes the i-th buyer's initial inventory position;
- $q_{i,1},\ldots,q_{i,k_0}$ denote the sizes of k_0 initial consecutive lots shipped at $t_1,\ldots,t_{k_0}$ by the vendor for the i-th buyer. The sizes of cumulative vendor's consecutive lots will be denoted by

$$q_j=\sum_{i=1}^{N}q_{i,j}=\lambda^j q_0 \quad \text{where } q_0=\sum_{i=1}^{N}q_{i,0} \text{ for } j=1,\ldots,k_0;$$

- $q_{i,k_0+1},\ldots,q_{i,k_0+k_i}$ denote the sizes of k_i consecutive last lots shipped at $t_{i,1},\ldots,t_{i,k_i}$ individually by the i-th buyer. The jth buyer's batch is equal to

$$q_{i,j}=\frac{M_i}{k_i} \quad \text{for } j=1,\ldots,k_i \text{ if only } k_i>0.$$

Additionally, if $k_0>0$ we have

$$q_{i,j}=\frac{D_i}{D}q_j=\lambda^j q_{i,0} \quad \text{for each } j=0,1,\ldots,k_0-1$$

and

$$q_{i,k_0}+M_i=(T-t_{k_0})D_i+q_{i,0}. \tag{1}$$

The agents' costs relaxed to a PDC schedule $q=[q_{ij}]$ are defined in the natural way:

$$v_i(q)=\begin{cases}\dfrac{1}{T}[A+k_0A_0+h_0\int_0^T I_0(t)\,dt] & \text{for } i=0,\\[2mm] \dfrac{1}{T}[k_iA_i+h_i\int_0^T I_i(t)\,dt] & \text{for } i>0,\end{cases} \tag{2}$$

where each $I_i(t)$ depends on technological parameters ξ and decision parameters $Q,\tilde{M}$ and $\tilde{k}$ of the PDC schedule.