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Competition under capacitated dynamic lot-sizing with capacity acquisition

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ABSTRACT

Lot-sizing and capacity planning are important supply chain decisions, and competition and cooperation affect the performance of these decisions. In this paper, we look into the dynamic lot-sizing and resource competition problem of an industry consisting of multiple firms. A capacity competition model combining the complexity of time-varying demand with cost functions and economies of scale arising from dynamic lot-sizing costs is developed. Each firm can replenish inventory at the beginning of each period in a finite planning horizon. Fixed as well as variable production costs incur for each production setup, along with inventory carrying costs. The individual production lots of each firm are limited by a constant capacity restriction, which is purchased up front for the planning horizon. The capacity can be purchased from a spot market, and the capacity acquisition cost fluctuates with the total capacity demand of all the competing firms. We solve the competition model and establish the existence of a capacity equilibrium over the firms and the associated optimal dynamic lot-sizing plan for each firm under mild conditions.

1. Introduction

One of the fundamental problems in operations management is determining the investment in capacity. A firm's capacity determines its maximal potential production. To acquire capacity is usually cost and time consuming, and once the investment is made, the cost is often partially or completely irreversible, as installed capacity is difficult to adjust in the short term. Moreover, the decision on how much capacity to acquire also strongly influences the action space for future operations planning. To invest in too much capacity wastes resources that could be used for other important operation activities, such as new product development and marketing; to invest in too little capacity means long waiting times, missed sales opportunities and lost revenue. Therefore, it is necessary to find an effective and comprehensive method to determine the proper capacity configuration for operations.

Increasing the capacity does not necessarily improve the operational performance, even if the product profit margins are large, because capacity acquisition cost is usually negative correlated to the production cost and often affected by the competitive resource environment. In addition, the competitors' other decisions, such as the timing of production and quantity, also affect capacity acquisition cost and investment performance. Game-theoretic modelling has been an effective method of describing and solving competition problems. In this paper, we solve a game-theoretic model of capacity competition problem over a finite-period planning horizon for a multiple-firm industry that uses a common resource to produce its products. For each firm, its best-response problem is a single-item capacity acquisition and lot-sizing problem.

The best-response problem considers a single-production facility that produces a single product item to satisfy a deterministic demand stream. The best-response problem for individual firms simultaneously determines an optimal capacity and a lot-size plan over the planning horizon. The capacity acquisition, production and inventory holding costs are considered. We formulate the problem as a cost minimizing Mixed Integer Non-Linear Programming (MINLP) model. This general problem class is impossible to solve using a polynomial time algorithm. Thus, we discretize the possible capacity choices and solve it for each of those. The major difference between the best-response problem and the classical capacitated lot-sizing problems is that the capacity level is an internal decision in our model.

Given the capacity competition model, we discuss the capacity equilibrium and associated optimal dynamic lot-sizing plans by analyzing the resulted best-response problem. We introduce an approximation for a firm's best-response function, showing through a numerical study that its use results in only a minor difference to the actual cost figures but still has desirable properties. We then proceed to analyze the competitive problem and show the existence of an equilibrium under modest assumptions. To the best of our knowledge, this is the first study to address

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lot-sizing problems considering resource competition. Moreover, since the complexity of the capacity competition problem, the approximated solutions are acceptable in practice.

The remainder of this paper is organized as follows. We review the relevant studies in Section 2. Section 3 introduces the relevant notation and the basic competitive model. Section 4 first describes the best-response problem that an individual firm faces when making its purchasing and lot-sizing decisions. In Section 5, we show our suggested solution in a structure of the game which results in an equilibrium following a standard procedure. Finally, a computational study and numerical examples are discussed in Section 6.

2. Literature review

The aim of capacity-acquisition decisions is to select the proper capacity that not only satisfies demand completely, but also minimizes the total capacity acquisition and lot-sizing cost. The research on capacity-investment problems includes two main streams, the traditional mathematical programming models and the economic models.

Traditional mathematical programming methods have been applied to capacity-acquisition problems ever since research efforts first took notice of them. The flexible capacity investment and management problems arose and were addressed at a relatively early stage. Fine and Freund (1990) present a two-stage stochastic programming model and an analysis of the cost-flexibility trade-offs involved in the investment in product-flexible manufacturing capacity for a firm. They address the sensitivity of the firm's optimal capacity-investment decision to the costs of capacity, demand distribution and risk level. Also, van Mieghem (1998) studies the optimal investment problem of flexible manufacturing capacity as a function of product prices, investment costs and demand uncertainty for a two-product production environment. He suggests finding the optimal capacity by solving a multi-dimensional news-vendor problem assuming continuous demand and capacity. Netessine et al. (2002) propose a one-period flexible-service capacity optimization and allocation model taking the capacity acquisition, usage, and shortage costs into account. While each paper considers the multiple products and multiple resource problems with demand uncertainties, their focus is limited to single-period models.

Apart from the studies which focus on flexible capacity investment, many efforts to solve generalized capacity-investment problems have also been made. Harrison and van Mieghem (1999) develop a single-period planning model to incorporate both capacity investment and production decisions for a multiple-product manufacturing firm. Their study yields a multi-dimensional descriptive model generated from the "news-vendor model", and gives qualitative insights into real-world capacity-planning and capital-budgeting practices. Nevertheless, the decisions on optimal capacity investment are highly generalized, and the production plan decisions are not explicitly presented. van Mieghem and Rudi (2002) extend the work of Harrison and van Mieghem (1999) to include an operations environment with multiple products, production processes, storage facilities and inventory management. Moreover, they investigate how the structural properties of a single period extend to a multi-period setting. They also improve previous studies by considering some inventory-management issues.

Many studies have made extensive use of game-theoretic models in the development of product pricing and competitive strategic investment models, among others. For instance, van Mieghem (1999) uses a game-theoretic approach to model the coordination process of simultaneous investment, production, and subcontracting decisions. The model's objective is to maximize the overall supply chain system profit and to analyze the size and timing of capacity investment. While capacity acquisition problems have been studied extensively, each paper mentioned above focused on single-firm operations. The competition for resources, however, is a common phenomenon in real-world operations in a multi-firm industry involving a particular product but is generally ignored in the literature because it often increases the intractability of the models, regardless of whether the model is stochastic or game theoretic.

Increasing global competition and cost pressure force businesses to discover undetected cost-saving potentials on investment in resources. Arnold et al. (2009) presents a deterministic optimal control approach optimizing the procurement and inventory policy of a company that is processing a raw material when the purchasing price, holding cost, and the demand rate fluctuate over time. However, they do not consider the effect of resource competition.

The three papers listed below address capacity decision problems emphasizing real-world capacity competition. Roller and Sickles (2000) propose a two-stage pricing and capacity-decision model considering price and capacity competition simultaneously. In the first stage, the capacity is determined and a price-setting game is performed in the second stage. Chen and Wan (2005) also study a service capacity competition problem for two make-to-order firms that are modelled as single-server queueing systems. They characterize the Nash equilibrium of the competition. The firms make their capacity choice based on the equilibria. Cheng et al. (2003) study the price and capacity competition of two application-service providers. The authors suggest that the providers with higher capacity would charge a higher price and enjoy a larger market share. Although capacity competition problems have not completely escaped notice, the aforementioned studies focus strictly on service industries modelled as queueing systems. The special operations nature of the service industry restricts the methods from being generalized to other industries, such as manufacturing or other more complicated service systems.

While great progress has been made in the development of capacity-investment models and approaches, most studies have focused on macro-analysis rather than practical applications. Many complicated decision factors, such as time-varying costs and inventory management, have been left unconsidered. In this sense, lot-sizing methods can compensate perfectly for this deficiency in the game-theoretic models, with the combination approach resolving the real-world capacity-investment and production problems more realistically.

Lot-sizing problems have been studied extensively for the past half century. Wagner and Whitin (1958) give a forward algorithm for a general dynamic version of the uncapacitated economic lot-sizing model. Since then, various variants, including single-item and multi-item, uncapacitated and capacitated lot-sizing problems, remain an important topic in Operations Research fields. More recent results include Federgruen and Tzur (1991), who consider a dynamic lot-sizing model with general cost structure. The authors give a simple forward algorithm which solves the general dynamic lot-size model in $O(T\log T)$ time and with $O(T)$ space requirement. This is an important improvement over the well-known shortest path algorithm solution in $O(T^2)$ space, advocated previously. Wagelmans et al. (1992) extend the range of allowable cost data to allow for coefficients that are unrestricted in sign. They developed an algorithm to solve the resulting problem in $O(T\log T)$ time.

However, the uncapacitated lot-sizing problem is an ideal case and hardly applicable to real-world operations. Capacity constraints always heavily influence production-plan decision