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# A new lot-sizing heuristic for manufacturing systems with product recovery

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ABSTRACT

We consider a deterministic model of the manufacturing system with product recovery. Two types of policies for the problem had been proposed in literature, namely the (1,$R$) policy, in which one manufacturing setup is followed by $R$ remanufacturing setups and the ($P$,1) policy, which has $P$ manufacturing setups, following every remanufacturing setup. Teunter (2004) developed heuristics to evaluate the cost for both policies and recommended choosing the better one among them. In this paper, we develop a new class of general ($P$,$R$) policies, where the long-run ratio of the number of manufacturing setups to the number of remanufacturing setups is $P/R$. Rather than have $P$ manufacturing setups followed by $R$ remanufacturing setups, we interleave (or intersperse) the setups of the manufacturing lots and the remanufacturing lots in such a way that the buildup of the recoverable inventory is minimized. We develop interleaving based ($P$,$R$) policy heuristics for the problem. Numerical results presented in the paper show that the proposed heuristic outperforms or performs as well as the best of the Teunter (2004) policies for all the problems tested.



## 1. Introduction

We consider a deterministic model of the manufacturing system with product recovery. The system manufactures new items as well as remanufactures the returned items. The demand rate for new items and return rate for the recoverable items are deterministic and constant. The relevant costs are holding costs for the new/serviceable items and recoverable items, and fixed setup costs for both manufacturing and remanufacturing. The problem is to determine the timing and the lot sizes for the manufacturing and remanufacturing setups, so as to minimize the long-run average cost per year.

Teunter (2004) had proposed two types of policies for the problem, namely the (1,$R$) policy, in which one manufacturing setup is followed by $R$ remanufacturing setups and ($P$,1) policy, which has $P$ manufacturing setups and one remanufacturing setup. Teunter develops heuristics to evaluate the cost of the (1,$R$) and ($P$,1) policy and recommends choosing the better one among the two policies. In the above policies, $P$ and $R$ are restricted to be integers to facilitate implementation, and this may lead to adjustment of the lot sizes and an increase in cost.

In this paper, we consider a general class of ($P$,$R$) policies. For every $P$ manufacturing setups over time, there will be $R$ remanufacturing setups. Rather than having $P$ manufacturing setups followed by $R$ remanufacturing setups, we interleave (or intersperse) the setups of the manufacturing lots and remanufacturing lots in such a way that the buildup of the recoverable inventory is minimized. We develop two interleaving based ($P$,$R$) policy heuristics for the problem. In the first heuristic, the manufacturing lot size, $Q_p$, as well as the remanufacturing lot size, $Q_r$, is always kept fixed over time. In the second heuristic, we either allow the lot size for remanufacturing, $Q_r$ [when ($P>R$)] or the lot size for manufacturing, $Q_p$ [when ($P<R$)] to vary in different setups in a cycle. We then compare and numerically evaluate the proposed heuristics with the best of the (1,$R$) and ($P$,1) heuristic.

The rest of the paper is organized as follows. We first provide a review of the related literature in the remainder of this section. In Section 2, we give the notation and analyze the model. The first interleaving heuristic with non-varying lot sizes $Q_p$ and $Q_r$ is developed in Section 3. Section 3 also provides the numerical results for this heuristic. In Section 4, we develop the heuristic with variable remanufacturing (or manufacturing) lot size for the case when $P>R$ ($P<R$). Finally, Section 5 provides a summary and a few concluding remarks.

One of the first papers to consider the lot-sizing and scheduling problem in manufacturing systems with the product recovery is Schrady (1967). The production for manufacturing and remanufacturing is assumed to occur at an infinite rate. For this model, Schrady proposed the (1,$R$) policy, where every manufacturing setup is followed by an integer number, $R$, of identical remanufacturing setups. The order quantity for manufacturing and remanufacturing was kept stationary. Schrady's model was

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extended for finite product recovery rate by Nahmias and Rivera (1979). They studied a deterministic model with infinite production rate similar to that of Schrady, but with a finite recovery rate that is greater than the demand rate. They proposed a class of heuristic policy that is similar to the Schrady's. Koh et al. (2002) extended the work of Nahmias and Rivera (1979) by allowing remanufacturing or recovery rate to be either larger or smaller than the demand rate. Again as in Nahmias and Rivera (1979) and Schrady (1967), they consider only the class of (1,*R*) policy.

Teunter (2001) extended the work of Schrady (1967) to consider two classes of control policies: (*i*) the (1,*R*) policy, where every manufacturing setup is followed by an integer number, *R*, of remanufacturing setups, and (*ii*) the (*P*,1) policy, where every remanufacturing setup is followed by an integer number, *P*, of manufacturing cycles. In either class of policies, the order quantities or lot sizes used for manufacturing (and remanufacturing) are identical across different cycles. For these restricted classes of policies and with the relaxation that *R* (*P*) can be non-integer, Teunter derived the optimal solution as a closed form expression. However, as the number of remanufacturing (and manufacturing) setups has to be an integer, a heuristic solution is derived from his closed form expression. Also unlike Schrady (1967), the holding cost rates for the recoverable and serviceable items are assumed to be different.

In Teunter (2001), the manufacturing and remanufacturing rates were assumed to be infinite. Teunter (2004) extended the work of Teunter (2001) to consider finite production rates for the manufacturing and remanufacturing. Again two classes of policies are considered, and within these classes of policies, the optimal solution to the relaxed problem (with non-integer manufacturing or remanufacturing setups) is obtained. From this, heuristic solutions are obtained for these classes of policies. Konstantaras and Papachristos (2006) extended the work of Teunter (2004) by allowing for complete backordering of demand; however, they also consider only the restricted classes of (1,*R*) and (*P*,1) policies.

The above papers assume that all the returned items are remanufactured. There is an another set of papers that allow for a disposal option (wherein a certain proportion of the returned items can be disposed off instead being remanufactured). The focus of these papers is more on determining the optimal disposal rate. Papers that focus on the disposal option include Richter (1996a, 1996b), Richter and Dobos (1999) and Dobos and Richter (2004).

## 2. Notation and model analysis

The notation used throughout the paper is given in Table 1. We are considering a deterministic lot-sizing and scheduling model for a manufacturing system with product recovery. The system faces continuous external demand for a product at a constant rate $d$. The demand for the product can be satisfied with newly manufactured products or with products recovered or remanufactured from old, returned products. The remanufactured products are considered as good as new. The product return takes place at a constant rate $fd$, where $f<1$ is the return fraction. It is assumed that all the returned products can be recovered and remanufactured, and that there is no disposal of product returns without recovery. The production for manufacturing new products occurs at a continuous rate $p$, and the production for remanufacturing the recoverable items occurs at a continuous rate $r$. Both $p$ and $r$ are greater than the demand rate $d$. The inventory of manufactured or remanufactured products or *serviceable* items incur holding cost at rate $h_s$. The inventory of *recoverable* items (returned products that have not yet been remanufactured) incur holding cost at rate $h_r$. The setup cost per lot size for manufacturing the items is $K_p$ and the setup cost per lot size for remanufacturing the items is $K_r$. The objective is to find lot sizes and production schedule for manufacturing and remanufacturing that minimize the long-run average cost (comprised of production setup costs and inventory holding costs).

**Table 1**
Notation.

| | |
|---|---|
| $d$: | demand rate per year (i.e. total annual demand) |
| $f$: | return fraction (return rate $fd$) |
| $p$: | rate of production for manufacturing (of new items) |
| $r$: | rate of production for remanufacturing or recovery |
| $K_p$: | setup cost per production lot for manufacturing |
| $K_r$: | setup cost per lot for remanufacturing |
| $h_r$: | holding cost per recoverable item per year |
| $h_s$: | holding cost per serviceable item per year |
| $Q_p$: | production lot-size (or lot size for manufacturing) |
| $Q_r$: | recovery lot-size (or lot size for remanufacturing) |
| $\hat{P}$: | average number of manufacturing setups per year. Note that $\hat{P}$ need not be an integer |
| $\hat{R}$: | average number of remanufacturing setups per year. Note that $\hat{R}$ need not be an integer |
| $P$: | number of manufacturing setups for every $R$ remanufacturing setups. Note that $P$ and $R$ are integers |
| $D$: | Total demand in a time unit, e.g. one year |
| $TC^{(P,R)}(Q_p,Q_r)$: | total cost for ($P$,$R$) policy with lot sizes $Q_p$ and $Q_r$ |

As there is no disposal of returns, if the production lot size per manufacturing setups is $Q_p$, the average number of manufacturing setups per year is

$$\hat{P}=\frac{D(1-f)}{Q_p} \tag{1}$$

Similarly, if the lot size per setup for remanufacturing is $Q_r$, the average number of remanufacturing setups per year is

$$\hat{R}=\frac{Df}{Q_r} \tag{2}$$

As the ratio of number of manufacturing setups to remanufacturing setups per unit time is $P/R$, from Eqs. (1) and (2)

$$\frac{P}{R}=\frac{\hat{P}}{\hat{R}}=\frac{D(1-f)Q_r}{DfQ_p}=\frac{(1-f)Q_r}{fQ_p} \tag{3}$$

Note that for a general ($P$,$R$) policy, the ratio of $P/R$ (or $R/P$) do not have to be integers, even though $P$ and $R$ are integers.

For both ($P$,1) policy and (1,$R$) policy, the total cost can be derived without any difficulty as shown in Teunter (2004). In the policy that we propose in this paper, both $P$ and $R$ could be greater than 1. That is, for every $P$ manufacturing setups, there are $R$ remanufacturing setups. Unlike the ($P$,1) policy or (1,$R$) policy, the scheduling of the lot sizes is not straightforward here. One could of course have $P$ manufacturing setups followed by $R$ remanufacturing setups. But this would result in too much buildup of the returns or the recoverable inventory. Therefore, we suggest interleaving (or interspersing) the setups of the manufacturing lots and the remanufacturing lots in such a way that the buildup of the recoverable inventory is minimized.

## 3. Interleaving heuristic for general (*P*,*R*) policy

Assume that we will start a new manufacturing or remanufacturing setup only when the serviceable inventory is zero[1]. Let $INV_r$ represents the average recoverable inventory level for a particular policy. Then, clearly the total cost function of the ($P$,$R$) policy is

$$TC^{(P,R)}(Q_p,Q_r)=\frac{K_pd(1-f)}{Q_p}+\frac{K_rdf}{Q_r}+\frac{1}{2}h_s((1-f)(1-d/p)Q_p+f(1-d/r)Q_r)+h_rINV_r \tag{4}$$

[1] Note that such a zero inventory policy need not necessarily be optimal, especially if the holding cost rate of the recoverable inventory, $h_r$, is very high.