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Economic lot-sizing and dynamic quantity competition

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ABSTRACT

We study a problem of dynamic quantity competition in continuous time with two competing retailers facing different replenishment cost structures. Retailer 1 faces fixed ordering costs and variable procurement costs and all inventory kept in stock is subject to holding costs. Retailer 2 only faces variable procurement costs. Both retailers are allowed to change their sales quantities dynamically over time. Following the structure of the economic order quantity (EOQ) model, retailer 1 places replenishment orders in batches and retailer 2 follows a just-in-time (JIT) policy. The objective of both retailers is to maximize their individual average profit anticipating the competitor's replenishment and output decisions. The problem is solved by a two-stage hierarchical optimization approach using backwards induction. The second-stage model is a differential game in output quantities between the two retailers for a given cycle length. At the first stage, the replenishment policy is determined. We prove the existence of a unique optimal solution and derive an open-loop Nash equilibrium. We show that both retailers follow contrary output strategies over the order cycle. The EOQ retailer, driven by inventory holding costs, decreases his market share whereas the output of the JIT retailer increases. Moreover, depending on the cost structure, the EOQ retailer might partially be a monopolist. At the first stage, the EOQ retailer determines the cycle length, anticipating the optimal output trajectories at the second stage.

1. Introduction

The goal "matching supply with demand" which is the fundament of *supply chain management* has changed the role of operations from pure cost control to value creation. In recent years, the coordination of supply and demand management has received considerable attention. In particular, determining the "right" selling price is a complex task which not only requires detailed demand information but also accurate information about the own operating costs, supply structure, and competitive environment. The benefits of *dynamic pricing* which is defined as the application of intertemporal price adjustments have long been known from *revenue management*.

An important research question is the choice of the right replenishment strategy. Firms may order just-in-time (JIT) without holding inventory, however, if each order is associated with fixed or setup costs, the firm orders in batches. The economic order quantity (EOQ) model optimizes the trade-off between fixed ordering costs and inventory holding costs. One of the fundamental assumptions is that the demand rate is constant which yields that the inventory level decreases linearly over an order cycle. However, in a monopolistic environment, it has been shown that if the retailer is allowed to change the selling price which leads to a changing demand rate, a linearly decreasing inventory level is not optimal (Rajan et al., 1992; Transchel and Minner, 2009).

There is a wide range of literature studying pricing and inventory/production control in a competitive environment. An overview over competition models in revenue management, where the common assumption is that capacity is fixed, perishable, and cannot be replenished, is provided by Talluri and van Ryzin (2004) and Bitran and Caldentey (2003). Elmaghraby and Keskinocak (2003) survey contributions that integrate dynamic pricing and inventory control. They distinguish models that consider a fixed and perishable capacity which cannot be replenished and models where inventory replenishment is incorporated. Since our model considers an inventory control and pricing problem where the replenishment option is integrated, we limit the following literature review to contributions that study dynamic inventory/production and pricing problems with replenishment option. Other surveys of existing contributions in this field are Chan et al. (2004) and Yano and Gilbert (2004).

Cost and market structure have a significant impact on operations and pricing decisions and the overall company performance. Gaimon (1989) analyzes a differential game of two competing retailers who choose price and capacity where the acquisition of new technology reduces a firm's unit operating

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costs. An open-loop and a closed-loop Nash equilibrium are derived for optimal price and capacity strategies of both retailers. She shows that the dynamic Nash strategies obtained from the closed-loop model exhibit a more restrictive new technology acquisition and a greater reduction of existing capacity relative to the open-loop one. Furthermore, the prices that are charged in a closed-loop Nash equilibrium are higher compared to an open-loop Nash equilibrium strategy. Eliashberg and Steinberg (1991) study the problem of determining production and marketing equilibrium strategies of two competing firms under dynamically changing demand. The firms differ with respect to their production costs. One firm faces convex production costs and linear holding costs whereas the second firm faces linear production costs and will not hold inventory. Both firms are allowed to vary their production rates and their selling prices continuously over a finite planning horizon. They characterize the open-loop equilibrium strategy of the two firms and show that the firm with convex production costs will build up inventory at the beginning of the planning horizon, then continue by drawing down inventory until it reaches zero, and finally, the firm will follow a "zero-inventory" policy until the end of the planning horizon. Min (1992) extends the profit-maximizing economic order quantity model to the case of a symmetric oligopoly and derives economic implications regarding selling prices, demand elasticities, number of competitors, marginal and average costs, as well as average holding costs. Lederer and Li (1997) study a competition problem where firms compete for customers by setting prices, production rates for each type of customers, and a production schedule where the customers are sensitive regarding price and delay time. Customers are either homogenous or differ with respect to their demand function and delay sensitivity. They show the existence of an equilibrium which is uniquely defined if the firms can differentiate between their customers. Cachon and Harker (2002) analyze an economic order quantity game between two retailers with fixed ordering costs and price-sensitive consumers. They investigate outsourcing if there exists a supplier who is able to manage either firm's operations and charges a constant fee per unit of demand for that service. They further show that there exist contracts that give the supplier a positive profit and yield a higher profit to either retailer than if the retailers do not outsource. Ha et al. (2003) analyze the role of delivery frequency in supplier competition. They examine how the nature of competition (price or delivery) and the decision rights (who is responsible for handling logistics, pricing, and delivery) influence supply chain performance. When suppliers compete through prices, higher delivery frequencies may result in more intensive price competition, which is beneficial to the customers. Kogan and Tapiero (2007) consider both discrete and continuous time intertemporal models of competition and coordination in supply chains. For various supply chain problems the authors illustrate the effect of dynamic conditions on supply chain performance when decisions can be made at any point in time. Adida and Perakis (2010) investigate a manufacturing system where two firms with different production capacity compete through pricing and inventory control. They study a Nash equilibrium for the decentralized game and determine the optimal solution when a central authority controls the entire system. Beside some intuitive results, e.g., that in both settings, the firm with lower production capacity charges higher prices, produces less, and generates less profit compared to the firm with higher production capacity, they show that in the decentralized setting, the firm with the lower capacity may want to restrict its capacity even when additional capacity is available at zero costs. Models that consider fixed ordering costs and non-stationary demand are typically based on a discrete time framework. Federgruen and Meissner (2009) develop a competitive pricing model for a problem with time-varying fixed and variable procurement costs. They establish the existence of a price equilibrium and the associated optimal dynamic lot-sizes. Furthermore, they design efficient procedures to compute the equilibrium prices and dynamic lot-sizing plans.

Models that incorporate fixed ordering costs in a continuous time model, i.e., the points in time of decision making are not predetermined, are rather rare. This paper focuses on the interaction of a dynamically changing sales quantity and the replenishment policy in a competitive environment. The traditional economic order quantity (EOQ) problem is extended to the case of two competing retailers who might vary the sales quantity continuously over time. We examine the role of the replenishment policy in a deterministic continuous time Cournot quantity competition. The retailers differ in their respective cost structures. A replenishment of retailer 1 is subject to fixed ordering costs, variable procurement costs, and all inventory kept in stock is subject to holding costs. Retailer 2 only faces variable procurement costs. Following the assumptions of the EOQ model, retailer 1 places replenishment orders in batches whereas retailer 2 follows a just-in-time (JIT) policy. The objective of both retailers is to maximize their individual average profits taking the competitor's decision into account.

The problem is solved by a two-stage hierarchical optimization model. Given a fixed cycle length, we derive an open-loop Nash equilibrium for the second-stage differential game and compare the equilibrium output strategies of both retailers. At the first stage, the replenishment policy and therefore the optimal cycle length of the EOQ retailer is determined anticipating the optimal output trajectories at the second stage. We show participation conditions of both retailers and prove the existence and uniqueness of an optimal solution. The output decisions are not only influenced by the competitor's output but also by the current inventory level of retailer 1. Since the inventory level decreases over an order cycle, optimality conditions change continuously. Both retailers follow contrary output strategies over an order cycle. While retailer 1 decreases his market share over an order cycle, retailer 2 increases the market share over an order cycle, i.e. high inventories at the beginning of a cycle induce more aggressive output strategies. Furthermore, we show that for certain cost parameters, the retailers do not necessarily compete over the entire order cycle.

The paper is organized as follows. In Section 2, we present the competition and procurement model as a differential game. In Section 3, we analyze the problem by solving a hierarchical two-stage optimization model. We show the participation condition of both retailers and derive structural properties of the optimal solution. Finally, we provide concluding remarks and a brief outlook for future research in Section 4.

2. Model formulation

Consider two retailers competing on a common sales market over an infinite planning horizon where each of them seeks to maximize its average profit. The average profit equals revenue minus procurement costs whereas both firms differ in their respective cost structure. A replenishment of retailer 1 is subject to fixed ordering costs F_1 and variable procurement costs c_1 per unit. Due to the fixed cost, retailer 1 purchases in batches and holds inventory that is subject to holding costs h_1 per unit and time unit. Retailer 2, on the other hand, only faces variable procurement costs c_2 per unit so that retailer 2 has no incentive to batch and follows a JIT strategy. Following the assumptions of the EOQ model, retailer 1 places replenishment orders in batches of size Q every T periods.