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Optimal lot sizing in a non-cooperative material manager–controller game

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ABSTRACT

It is important for manufacturing companies to optimise purchase order quantities. Inaccurate lot size planning raises costs and lowers profits, which top management of course attempts to avoid through controlling processes. The lot size decision becomes even more relevant in the case of just-in-time delivery within a supply chain.

The interaction between lot sizing and auditing can be described in terms of a modified inspection game. This paper considers how probabilities, which are the basis for the mixed strategies at equilibrium in the inspection game, will change if the level of penalties accruing to the two players (material manager and controller) depends on the cost deviation caused by the material manager's poor lot-size planning. It is evident that the Nash equilibrium shifts to the strategy combination (methodically determined decision and low auditing level), if the penalties imposed on the controller and material manager increase.

Penalties that depend on such deviations, and an accurate audit of the controller's report by top management, prove to be the best instruments for avoiding mismanagement by the material manager and poor controller work, both of which lead to high costs.

1. Introduction

In this paper, the efficient design of auditing lot size planning, operated by a material manager and audited by a controller, will be discussed on the basis of a game theoretic approach. Especially, the extent to which a large cost deviation ΔK, due to non-optimal lot size planning, influences the Nash equilibrium of the two players' decisions will be analysed.

Stadtler (2007) comes to the conclusion, that, based on the classic order-quantity formula of Harris (1913), the impact of a sub-optimal order quantity, taking into account the overall cost situation, is low, and that a given bound ε is only exceeded if there are large quantity deviations δ. That means, that for Stadtler (2007) it is not so important to determine the optimal lot size, because the total cost function in the vicinity of the Economic Order Quantity (EOQ) is relatively flat (see K^{EOQ} in Fig. 1). However, other studies demonstrate that cost deviations caused by an erroneously determined order quantity in the case of just-in-time (JIT), may indeed be substantial (Fandel et al., 1988; Silver, 1992). This can be attributed in part to the fact that the same level of quantity deviations, with substantially reduced optimal order quantities, are more significant and conspicuous. In addition, in the case of just-in-time, the total cost function is steeper, due to the substantial significance of higher inventory costs (see K^{JIT} in Fig. 1). Therefore, the buyer has to defray the cost deviations, if he departs from the optimal lot size.

This decision-making situation becomes even more intense, because optimal order quantities, which are determined according to the just-in-time principle, need to be maintained strictly in supply chains, in order to ensure that the entire value-creation process within the supply chain is really maximised (Fandel and Stammen, 2004). Investigations of the interaction between supplier and buyer within supply chains, in the context of order quantities, include Reyniers (1992), Cachon and Netessine (2004) and Mileff and Nehez (2006). In these studies, game theory is applied as the problem-solving instrument. In contrast to models with coordinated inventory management within in the supplier–customer chain, such as van Houtum et al. (1996) for the case of central inventory planning, we take the perspective of an inspection game (see the fundamental work of Avenhaus et al., 2002) installed in the buyer. This means that a controller checks on the optimality of orders made by the manager. Simultaneously, the case is modelled, in which inadequate procedures on the part of both players will be revealed with a probability of p_a.

The parameter p_a does not have the quality of a strategic parameter, but implies an exogenous probability. The model also ensures that the controller, as well as the manager, will be punished if the controller audits the manager's work at a low level, and does not detect the non-methodical decision of the material manager, which is then subsequently detected by top management. The punishments for the controller and for the manager are described by $S_C = c_C \Delta K$ and $S_M = c_M \Delta K$, respectively. The parameter c_M symbolises the punishment coefficient of the material manager and c_C the controller's punishment

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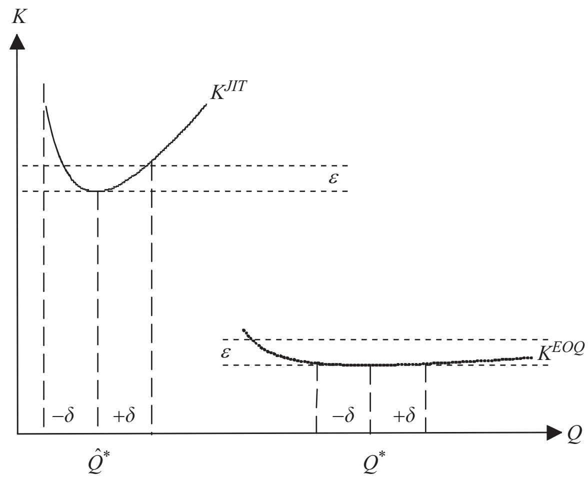

Fig. 1. Cost functions with EOQ and JIT.

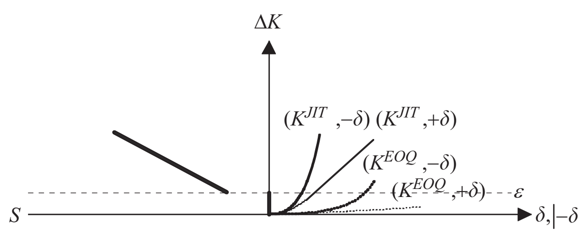

Fig. 2. Punishment for the controller and the manager in dependence on lot size and cost deviations.

coefficient. For the sake of simplicity and without any loss of generalisation, we assume further on that $c_C = c_M = 1$. These punishments will occur only if the cost deviation ΔK exceeds a given threshold ε (see Fig. 2). Simply, in this manner, the model takes the form of an inspection game relating to the nature of a lateral material manager–controller conflict. We show that, through our modification of the inspection game, the objective of maximising the company's profit will not be achieved, if top management evaluates work as correct, when it is in fact incorrect. In the following approach, we analyse the extent to which a large cost deviation, due to non-optimal lot-size planning, influences the Nash equilibrium of the two players' decisions. The modified inspection game is based on the approaches of Dresher (1962), Avenhaus et al. (1996) and Avenhaus et al. (2002). In business economics, the inspection game was formulated by Borch (1982), who describes the behaviour of an accounting clerk playing against a company. These considerations will now be applied to the problems associated with order-quantity planning, and then investigated.

2. The material manager–controller conflict as an inspection game

The game is described by the extensive-form game tree in Fig. 3.

In the following analysis, the two players "material manager" and "controller" each have two actions to choose between. The material manager (inspectee) can:

- calculate the lot sizes methodically (m) or
- plan them non-methodically according to instinct (nm) (following Fellingham and Newman (1985), who consider the case of an inspected manager).

In contrast, the controller (inspector) can control the materials manager:

- at a high audit level (h) or
- at a low audit level (nh) (following Ewert, 2004).

In total, the four pure strategies described below, which in principle, could form a Nash equilibrium, can appear in the decision-making situation modelled in this way:

- The material manager acts correctly by determining the lot size methodically and the controller also acts correctly, by exercising a high level of control with regard to the material manager–action combination (m, h).
- The material manager acts incorrectly by not determining the lot size methodically and the controller acts correctly by exercising a high level of control with regard to the material manager–action combination (nm, h).
- The material manager acts correctly by determining the lot size methodically and the controller acts incorrectly by not exercising a high level of control with regard to the material manager–action combination (m, nh).
- The material manager acts incorrectly by not determining the lot size methodically and the controller also acts incorrectly by not exercising a high control level with regard to the material manager–action combination (nm, nh)—whereby a split must be made into action combinations (nm, nh, a) and (nm, nh, na). Here, (a) designates the action that company management detects the sub-optimal behaviour of the material manager and the controller, and (na) that this remains undetected.

Subsequently, the following payoffs $\pi_n(M)$ for the material manager and $\pi_n(C)$ for the controller apply at the five end nodes $n\,(n=1,\ldots,5)$:

$$\pi_1(M) = p_h p_m (Z + B_M) \tag{1}$$

$$\pi_1(C) = p_h p_m (V - K) \tag{2}$$

$$\pi_2(M) = (1-p_h) p_m (Z + B_M) \tag{3}$$

$$\pi_2(C) = (1-p_h) p_m V \tag{4}$$

$$\pi_3(M) = p_h (1-p_m)(Z - S + L) \tag{5}$$

$$\pi_3(C) = p_h (1-p_m)(V - K + B_C) \tag{6}$$

$$\pi_4(M) = p_a (1-p_h)(1-p_m)(Z - S + L) \tag{7}$$

$$\pi_4(C) = p_a (1-p_h)(1-p_m)(V - S) \tag{8}$$

$$\pi_5(M) = (1-p_a)(1-p_h)(1-p_m)(Z + B_M + L) \tag{9}$$

$$\pi_5(C) = (1-p_a)(1-p_h)(1-p_m) V \tag{10}$$

The symbols in our game are described as follows.

For the material manager: Z is the basic salary, S the punishment, if the poor decision is detected (expressed as loss of reputation or monetary losses), B_D the bonus if the planning action of the material manager is certified as good work, without complaints from the controller, L the leisure profit, which the material manager will obtain, if he does not decide methodically, p_m the probability that the material manager assesses the optimal lot size as methodically correct.

For the controller: V is the basic salary, K the additional costs caused by a high auditing level, B_C the bonus if the controller detects insufficient planning by the material manager, S the punishment, if top management detects the low auditing level by the controller, who does not detect the non-methodically determined lot-size planning of