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A column generation heuristic for dynamic capacitated lot sizing with random demand under a fill rate constraint

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ABSTRACT

This paper deals with the dynamic multi-item capacitated lot-sizing problem under random period demands (SCLSP). Unfilled demands are backordered and a fill rate constraint is in effect. It is assumed that, according to the static-uncertainty strategy of Bookbinder and Tan [1], all decisions concerning the time and the production quantities are made in advance for the entire planning horizon regardless of the realization of the demands. The problem is approximated with the set partitioning model and a heuristic solution procedure that combines column generation and the recently developed ABC_β heuristic is proposed.

1. Introduction

We consider the stochastic version of the dynamic multi-item capacitated lot-sizing problem (CLSP). The problem is to determine production quantities to satisfy demands for multiple products over a finite discrete time horizon such that the sum of setup and holding costs is minimized, whereby a capacity constraint of a resource must be taken into consideration. In contrast to the deterministic CLSP, we assume that for every product k and period t the demand is a random variable D_{kt} ($k=1,2,\ldots,K$; $t=1,2,\ldots,T$). The period demands are non-stationary (to permit dynamic effects such as seasonal variations, promotions, or general mixtures of known customer orders with random portions of period demands), which usually is the case in a material requirements planning (MRP) based environment. Demand that cannot be filled immediately from stock on hand is backordered. As the precise quantification of shortage penalty costs which involve intangible factors such as loss of customer goodwill is very difficult, if not impossible, we assume that management has specified a target service level. In particular, we assume that the fill rate criterion (β service level) is in effect, as this criterion is very popular in industrial practice (see Tempelmeier [2]).

Industrial (MRP based) planning practice usually applies a forecasting procedure that provides a deterministic time series of expected future demands. Uncertainty is taken into consideration by reserving a fixed amount of inventory as safety stock (see Wortmann [3], Baker [4]). The amount of this reserve stock is usually computed with simple rules of thumb borrowed from stationary inventory theory, e.g. the standard deviation of the demand during the risk period is multiplied by a quantile of the standard normal distribution. In this way, it is almost impossible to meet targeted service levels. In addition, using time-independent safety stocks under dynamic conditions may result in significant cost penalties (see Tunc et al. [5]).

It is obvious that apart from the MRP-inherent neglect of limited capacities this widely used approach completely ignores the impact that lot sizes have on the absorption of risk. For example, in a case when due to high setup costs large lot sizes are used which cover the demands of many periods, it probably will be optimal to use no safety stock at all. On the other hand, if setup times or costs are reduced through technical measures in order to reduce lot sizes and the associated cycle stock, the required safety stock will increase.

In addition, which is even more problematic, the dynamic alteration of the materials requirements as a consequence of newly observed demand realizations according to the MRP planning process leads to random releases of production lots, as the actual timing and size of the required replenishments are the outcome of the demand process, which is random.

The resulting increase of the variance of the production quantities may have some unwanted consequences. First, in multi-level bill-of-material structures (or supply chains), the random change of a production order of a parent item leads to random requirements for its predecessors. This may cause the rescheduling of the production orders for the predecessors, which is a problem if a predecessor comes from an external supplier. If production orders are rescheduled, then demand variations occur

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that are propagated upstream through the supply chain, and which must be accounted for through buffers. In the literature this issue is discussed as planning nervousness. Second, the random change of the timing or size of a production lot directly translates into random resource requirements. For a machine, this is usually not a problem as long as the capacity of the machine is not overloaded. If an overload occurs, however, with fixed machine capacities this implies that the production plan becomes infeasible. In this case the planned due dates will be missed. This is one of the biggest problems found in short-term production planning in industry. In addition, there may even be cases when due to technical constraints the production quantities are unchangeable. This is often true in the process industries. Finally, if the considered resource is a human operator, then it may be unfavorable or even prohibited by labor agreement to change the workload in a period.

One countermeasure is the definition of a planning horizon with an unchangeable production plan (frozen schedule). This is what we study in the current paper. In the following, we assume that, according to the static-uncertainty strategy of Bookbinder and Tan [1], all decisions concerning the time and the production quantities are made in advance for the entire planning horizon, which is equivalent to using a frozen schedule. The unavoidable randomness of demand is accounted for through the appropriate sizing of the orders. Other than Bookbinder and Tan [1], we consider multiple products, a resource with limited capacity and a fill rate constraint.

The rest of this paper is organized as follows. In Section 2 the relevant literature is reviewed. Next, in Section 3, the considered stochastic lot-sizing problem under a fill rate per cycle constraint as proposed by Tempelmeier and Herpers [6] is approximated with a set partitioning model. Then, in Section 4, we present a heuristic column generation procedure to solve the LP-relaxation of this model and combine this procedure with the ABC_β heuristic proposed in Tempelmeier and Herpers [6] to solve the complete problem. The results of a numerical experiment are reported in Section 5. Finally, Section 6 contains some concluding remarks.

2. Literature

The deterministic multi-item dynamic capacitated lot-sizing problem has been studied for a long time. For recent overviews see Karimi et al. [7], Jans and Degraeve [8], Robinson et al. [9] and Buschkühl et al. [10]. However, only a limited number of researchers have considered dynamic capacitated lot sizing under random demand. A literature overview is presented in Sox et al. [11]. Sox and Muckstadt [12] solve a variant of the stochastic dynamic CLSP, where item- and period-specific backorder costs as well as extendible production capacities are considered. These authors propose a Lagrangean heuristic to solve the resulting non-linear integer programming problem that is repeatedly applied in a dynamic planning environment. Martel et al. [13] develop a branch-and-bound procedure for the solution of a similar model formulation.

Brandimarte [14] considers the stochastic CLSP where the uncertainty of the demand is represented through a scenario tree. In this case, the period demands are modeled as discrete random variables. The evolution of demand over time is depicted with a directed layered tree, where each layer corresponds to a planning period and the nodes are linked to realizations of the discrete stochastic demand process. The resulting large-scale deterministic MILP model is then solved with a commercially available solver using rolling schedules with lot-sizing windows. As demonstrated by Brandimarte [14], the scenario-based approach suffers from a dramatically increasing complexity, if the number of periods and/or the number of possible outcomes of the period demands are increased. In addition, currently there are no scenario-based models available which could account for product-specific fill rate constraints.

Tempelmeier and Herpers [6] propose a formulation of the dynamic capacitated lot-sizing problem under random demand, when the performance is measured in terms of a fill rate per cycle which is a popular performance measure in industry. They propose the ABC_β heuristic which is an extension of the $A/B/C$ heuristic of Maes and Van Wassenhove [15].

3. Problem formulation

Below the following notation is used:

b_t	capacity in period t (time units)
β_k^{*}	target fill rate per cycle for product k
c_n	total cost of production plan n of product k
D_{kt}	demand for product k in period t
F_{kt}	backorders of product k in period t
δ_n	binary selection variable for plan n
γ_{kt}	binary setup indicator for product k in period t
h_k	inventory holding cost per time period per unit of product k
I_{kt}	net inventory for product k at the end of period t
$I_{kt}^{f,\mathrm{end}}$	backlog of product k at the end of period t
$I_{kt}^{f,\mathrm{prod}}$	backlog of product k after production in period t, but before demand occurrence
K	number of products
κ_{nt}	capacity requirement of production plan n in period t
l_{kt}	number of periods since the last setup (product k, period t)
M	sufficiently large number
ω_{kt}	indicator variable: $\omega_{kt}=1$, if production of product k takes place in period $t+1$; $\omega_{kt}=0$ otherwise
$\mathcal{P}_k$	set of production plans for product k
π_t	dual variable associated to the capacity requirement constraint of period t
q_{kt}, q_{nt}	lot size of product k (production plan n) in period t
σ_k	dual variable associated to the plan selection constraint for product k
s_k	setup costs for product k
tb_k	capacity usage for production of one unit of product k
T	length of the planning horizon
$[x]^+$	$\max\{0,x\}$
$[x]^-$	$\min\{0,x\}$

Consider K products that are produced to stock on a single resource with given period capacities b_t $(t=1,2,\ldots,T)$. The planning situation is completely identical with that assumed in the classical dynamic capacitated lot-sizing problem (CLSP) without setup times. However, there is one exception: For each product k and each time period t, the period demands D_{kt} are random variables with a known probability distribution and given period-specific expected values $E\{D_{kt}\}$ and variances $V\{D_{kt}\}$. These data, which in a dynamic planning environment vary over time, are the outcome of a forecasting procedure. The demands of the various products are mutually independent and there is no autocorrelation. Unfilled demands are backordered. The amount of backorders is controlled by imposing a fill rate per cycle constraint. We define the fill rate per cycle as the ratio of the expected demand observed during the coverage time of a production order that is routinely filled from available stock on hand and the actual lot size. More precisely, let τ be a production period of