Int. J. Production Economics 136 (2012) 255–265

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Single-stage formulations for synchronised two-stage lot sizing and scheduling in soft drink production

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ARTICLE INFO

Article history:
Received 16 May 2011
Accepted 23 November 2011
Available online 13 December 2011

Keywords:
Production lot-scheduling models
Asymmetric travelling salesman problem
Soft drink industry

ABSTRACT

This study deals with industrial processes that produce soft drink bottles in different flavours and sizes, carried out in two synchronised production stages: liquid preparation and bottling. Four single-stage formulations are proposed to solve the synchronised two-stage lot sizing and scheduling problem in soft drink production synchronising the first stage's syrup lots in tanks with the second stage's soft drink lots on bottling lines. The first two formulations are variants of the General Lot Sizing and Scheduling Problem (GLSP) with sequence-dependent setup times and costs, while the other two are based on the Asymmetric Travelling Salesman Problem (ATSP) with different subtour elimination constraints. All models are computationally tested and compared to the original two-stage formulation introduced in Ferreira et al. (2009), using data based on a real-world bottling plant. The results show not only the superiority of the single-stage models if compared to the two-stage formulation, but also the much faster solution times of the ATSP-based models.

1. Introduction

This paper considers a production lot sizing and scheduling problem encountered in a soft drink bottling plant. Although the models developed are based on that plant, its production processes are sufficiently similar to those in many other bottling plants worldwide and even in other industries for the models proposed in this paper to be widely applicable. Integrated lot sizing and scheduling models have been researched in the context of real-word problems (Clark et al., 2011), for example, packaging company production yogurt (Marinelli et al., 2007), foundries (Araujo et al., 2008), electro-fused grains (Luche et al., 2009), glass container industry (Almada-Lobo et al., 2008), animal feed production (Toso et al., 2009), soft drink production (Ferreira et al., 2010), pharmaceutical company (Stadtler, 2011), sand casting operations (Hans and Van de Velde, 2011). Besides the real problem theoretical models have been extensively studied in the last years (Fleischmann, 1990; Haase, 1994; Drexl and Haase, 1995; Fleischmann and Meyr, 1997; Drexl and Kimms, 1997; Kang et al., 1999; Haase and Kimms, 2000; Meyr, 2000, 2002; Gupta and Magnusson, 2005; Fandel and Stammen-Hegene, 2006; Tempelmeier and Buschkühl, 2008; Gicquel et al., 2009; Kaczmarczyk, 2011).

Production lot sizing and scheduling problems can be very difficult depending on the restrictions which have to be met and on the combinatorial structure (classified in general as NP-hard optimization problems, e.g., Meyr, 2002; Bitran and Yanasse, 1982). In general the integrated lot sizing and scheduling problems are based on lot sizing models (Karimi et al., 2003; Toledo and Armentano, 2006; Helber and Sahling, 2010) adapted to incorporate the lot sequences. The sequence of lots involves the determination of when each lot is produced.

Different characteristics have been considered in the lotsizing and scheduling models. For example, the sequence dependent setup times and costs was studied by Fleischmann and Meyr (1997), Haase and Kimms (2000), Meyr (2000), Beraldi et al. (2008), and Kovàcs et al. (2009). The sequence-dependent setup costs and setup times with setup carryover problem was studied in Gupta and Magnusson (2005) and Menezes et al. (2011); Almeder and Almada-Lobo (2011) study the synchronisation in lot sizing and scheduling problems; Supithak et al. (2010) treat lot sizing and scheduling problems with earliness tardiness and setup penalties; Mateus et al. (2010) apply decomposition methods and an iterative approach for the integration of the problems; Stadtler (2011) studies multilevel lot sizing and scheduling

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doi:10.1016/j.ijpe.2011.11.028

problems with zero lead times. For reviews in lot sizing and scheduling problems the reader is referred to, e.g., Drexl and Kimms (1997), Koçlar (2005), Zhu and Wilhelm (2006), Jans and Degraeve (2008), and Robinson et al. (2009).

The formulations for lot sizing and scheduling problems can be mainly classified into two groups: small bucket and big bucket models. In small bucket models, such as the DLSP (Discrete Lot Sizing Problem, Fleischmann, 1990; Gicquel et al., 2009), the planning horizon is broken down into relatively small intervals in which at most one item can be produced. The sequence in small bucket formulations is inherent of the model. In situation in which there are many lots (periods), the total number of variables and constraints can increase significantly. In big bucket models, on the other hand, multiple items can be produced in each period. The strategy to incorporate the sequence in the model can be, for example, adding ATSP constraints (Menezes et al., 2011). An advantage is that the total number of variables and constraints is smaller.

An interesting formulation is the GLSP (General Lot Sizing and Scheduling Problem, Fleischmann and Meyr, 1997), in which the planning horizon is broken down into macro-periods and multiple items can be produced in each macro-period. However, to incorporate the sequence, each macro-period is divided into micro-periods in which at most one item can be produced, so its special structure involving subperiods within time periods may be associated with a small bucket framework (Koçlar, 2005). Clark et al. (2010) take a different approach using an asymmetric travelling salesman problem (ATSP) representation for sequencing lots rather than a GLSP-type model, obtaining good results. Although their formulation was inspired by the animal feed production case, the same idea is applicable to soft drinks production.

An important characteristic of soft drink production processes is the synchronisation between its two stages. This is necessary in case the start of production of lots at the second stage (drink bottling) depends on the lots at the first stage (syrup preparation). Toledo et al. (2007, 2009) propose a general model that synchronises the schedules of the soft drink plant's two production stages. Nevertheless, the mathematical model is rather complex, which has motivated the authors to develop approximate methods. An alternative model to represent a synchronised two-stage multi-machine problem is formulated in Ferreira et al. (2009). The authors simplify the overall problem by dedicating bottling lines to tanks.

This paper introduces alternative formulations for the lot sizing and scheduling problem in which the synchronised two-stage problem is formulated as a single-stage model. The first two formulations (models R1 and R2) are based on the single-stage GLSP model with sequence-dependent setup times and costs, while the other two are ATSP-based formulations (models F1 and F2) with different subtour elimination constraints.

In Section 2, we briefly explain the soft drink production process and summarize the synchronised two-stage formulation presented in Ferreira et al. (2009). In Section 3, the single-stage models R1 and R2 are presented, then Section 4 formulates the two models F1 and F2. Section 5 develops the solution procedures to solve the models. In particular, two strategies are detailed for solving model F2, based on the generation of subtour elimination inequalities and patching procedures. The models are computationally tested and analysed in Section 6. Concluding remarks and perspectives for future research are discussed in Section 7.

2. The soft drink production process

The soft drink production process has two main stages: flavour preparation (stage 1) and bottling (stage 2), as shown in Fig. 1. In stage 1, the liquid flavour (concentrated syrup plus some water) is prepared in tanks of varying capacities. Two different flavours cannot be prepared simultaneously in the same tank. For technical reasons a tank must be empty before a new lot of liquid flavour can be prepared in it, even if the flavour does not change. The preparation (cleaning) times and costs depend on the sequence of flavours. A minimum quantity of liquid flavour must be prepared in order to assure homogeneity as the tank propeller has to be completely covered in order to properly mix the necessary ingredients.

In stage 2, the liquid flavours are bottled at the filling lines. A filling line consist of a conveyor belt and machines that wash the bottles, fill them with a combination of liquid flavour and more water (carbonated or non-carbonated) and then seal, label and pack the filled bottles. If a bottle needs to be removed from the conveyor belt, then this is done at the end of the production process, before packaging. There is only one entry point for the bottles in the filling line. Conceptually, we can consider the entire filling line as a single machine processing items characterized by different flavour/bottle-size combinations. Although the syrup preparation is denoted as stage I, the tanks are only freed to start a new syrup preparation once its liquids are completely bottled at stage II. As an example, the *Tank* 1 of Fig. 1 will be available to prepare other syrup once the *Line* 1 finishes the liquid bottling. Obviously, the line can only start the production in case the syrup is ready.

A line can receive a liquid flavour from only one tank at a time, no matter how many tanks are available. However, a tank can

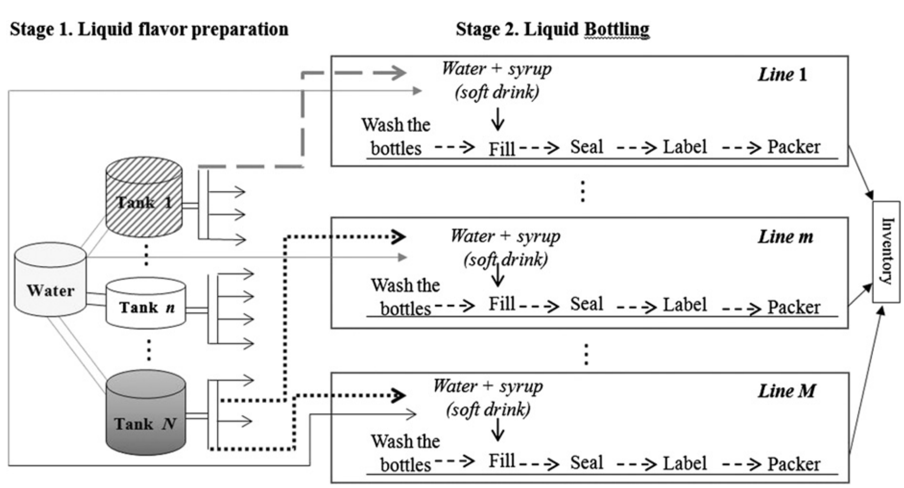

Fig. 1. Soft drink production process.