Int. J. Production Economics 137 (2012) 102–115

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Lot-sizing decisions for deteriorating items with two warehouses under an order-size-dependent trade credit

Jui-Jung Liao [a], Kuo-Nan Huang [b], Kun-Jen Chung [c,d,*]

[a] Department of Business Administration, Chihlee Institute of Technology, Taipei, Taiwan, ROC
[b] Department of International Business, St. John's University, Taipei, Taiwan, ROC
[c] College of Business, Chung Yuan Christian University, Chung Li, Taiwan, ROC
[d] National Taiwan University of Science and Technology, Taipei, Taiwan, ROC

ARTICLE INFO

Article history:
Received 18 September 2008
Accepted 11 January 2012
Available online 1 February 2012

Keywords:
Inventory
Deteriorating items
Two-warehouses
Trade credit

ABSTRACT

This study attempts to determine economic order quantity for deteriorating items with two-storage facilities (one is an owned warehouse and the other is a rented warehouse) where trade credit is linked to order quantity. As assumed herein, payment delays depend on the quantity ordered, when the order quantity is less than that at which a payment delay is permitted, the payment for the items must be made immediately. Otherwise, the fixed trade credit period is permitted. Furthermore, if the order quantity exceeds the owned warehouse capacity, it will be necessary to rent a warehouse which results in an additional rental cost. Otherwise, renting a warehouse is unnecessary. The problem discussed in this study involves how retailers decide whether to rent an additional warehouse to hold more items and thus obtain a trade credit period. First, a deterministic inventory model is developed for deteriorating items under the above situation. Second, this study demonstrates that the total cost function per unit time is convex via a rigorous proof. Third, five theorems are developed to optimize the replenishment cycle time and the order lot-size. Finally, numerical examples are used to illustrate these theorems and sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out and some important managerial insights are obtained.

1. Introduction

In recent decades, many studies have examined the problem of managing deteriorating items including medicines, volatile liquids, blood banks, foodstuffs and electronic components. Raafat (1991) presented a complete survey of the inventory literature on deteriorating inventory models. Moreover, Ghare and Schrader (1963) the first proponents proposed for developing a revised form of the EOQ model that assumed exponential decay. Covert and Philip (1973) then extended this model to consider the Weibull distribution deterioration. Other noteable works in this include those of Dave and Patel (1981), Sachan (1984), Hariga (1996) and their references. Recently, Goyal and Giri (2001) presented a review of the inventory literature published on deteriorating items since the early 1990s.

The conventionally adopted EOQ model assumes that the retailer must pay to purchase the item immediately upon receiving it from a supplier. However, such an assumption does not necessarily reflect the scenario in the real world. In fact, suppliers generally allow retailers' access to forward financing to increase demand or decrease inventory. This means that the supplier permits a trade credit period for the settlement of payment. The effect of the trade credit on the optimal inventory model has been examined in various studies. Goyal (1985) established an inventory model under permissible delay in payments. Shah (1993a,b) designed EOQ models for perishable items where payment delay is permissible. Other noteable works on this area were by Chand and Ward (1987), Aggarwal and Jaggi (1995), Chung and Liao (2006), Jamal et al. (2000), Chung (1998), Daellenbach (1986), Shinn (1997), Shinn and Hwang (2003), Liao (2007a,b), Zaid (2011), Ruo et al. (2011), Musa and Sani (2011), Tsao and Sheen (2012) and others. In fact, a key finding of these studies was that EOQ is independent of trade credit. Chung and Liao (2004) and Chang et al. (2003) considered the deteriorating items given the conditions of an order-size-dependent trade credit.

All the aforementioned inventory models implicitly assumed that the retailer owns a single warehouse with unlimited capacity. However, in more practical terms, any warehouse has a limited capacity. On the other hand, due to some reasons such as an attracted price discount for bulk purchase, the order costs higher than one using rented warehouse, and so on, inventory managers usually are attracted to hold more items than can be stored in an owned

* Corresponding author at: College of Business, Chung Yuan Christian University, Chung Li, Taiwan, ROC.
E-mail address: kjchung@cycu.edu.tw (K.-J. Chung).

0925-5273/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2012.01.020

warehouse. From this perspective, the two warehouse inventory models recently have been considered by various authors. This kind of system was first proposed by Hartely (1976). Sarma (1983) designed a deterministic inventory model with infinite replenishment rate and two storage levels. Furthermore, Murdeshwar and Sathe (1983) extended the case to incorporate finite replenishment rate. Other researchers that have studied in this area include Goswami and Chaudhuri (1992), Bhunia and Maiti (1998), Sarma (1987), Pakkala and Achary (1992a, 1992b), Benkherout (1997), Zhou (1998), Yang (2004) and Zhou and Yang (2005).

Due to the factors mentioned above, Chung and Huang (2006) considered a two-warehouse inventory problem for deteriorating item with limited storage space under permissible delay in payments. However, in certain practical situations, trade credits can be applied as an alternative to price discounts to order more quantities. Consequently, an important problem associated with inventory maintenance is deciding whether to rent an additional warehouse to hold more items to obtain a trade credit period.

Based on the above arguments, this study incorporates both Chung and Huang (2006) and Chung and Liao (2004) under above conditions. This study considers payment delay to depend on order quantity where the order quantity is less than that at which delayed payment is permitted, meaning payment must be made immediately. Otherwise, the fixed trade credit period is permitted. Additionally, if the order quantity exceeds owned warehouse capacity it becomes necessary to rent a warehouse which results in an additional rental cost. Given this marketing situation, this study develops a deterministic inventory model for deteriorating items with two warehouses (one is OW and the other is RW) and where trade credit is linked to order quantity. This study then demonstrates easy-to-use theorems to identify the optimal replenishment cycle time and the optimal order lot-size to minimize. Numerical examples are used to illustrate all of the study theorems and revealed the decision whether to rent an additional warehouse. Finally, sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out and some important managerial insights are obtained.

2. Notations and assumptions

We adopt the following notations for the model to be discussed:
Notations

C	unit purchase cost
S	ordering cost
A	rental cost for renting an additional warehouse
M	credit period set by the supplier
h	unit stock holding cost for item in OW (excluding capital opportunity cost)
k	unit stock holding cost for item in RW (excluding capital opportunity cost)
R	capital opportunity cost (as a percentage)
I	earned interest rate (as a percentage)
Q	order size
T	replenishment cycle time
D	annual demand rate
λ	a constant deterioration rate
$\overline{W}$	quantity at which the delay in payments is permitted
W	the storage capacity of OW
t_W	the time that inventory level reduce to W

$$T_{\overline{W}} = \frac{1}{\lambda}\ln\left(\frac{\lambda}{D}\overline{W}+1\right), \quad T_a = \frac{1}{\lambda}\ln\left(\frac{\lambda}{D}W+1\right)$$

In addition, we adopt the following assumptions for the model to be discussed:

(1) Replenishments are instantaneous with a known and constant lead time.
(2) No shortages are allowed.
(3) The demand rate is known with certainty and is uniform.
(4) The supplier proposes a certain credit period in paying for purchasing cost and the sales revenue generated during the credit period is deposited in an interest bearing account with rate I. At the end of the period, the credit is settled and the retailer starts paying the capital opportunity cost for the items in stock with rate $R (R \geq I)$.
(5) The daily expenses of the system can be overcome from the difference between retail price and unit cost.
(6) The time to deterioration of each item follows an exponential distribution with parameter λ, and the deteriorated units are not replaced.
(7) If $Q < \overline{W}$, the delay in payments is not permitted. Otherwise, certain fixed trade credit period M is permitted.
(8) The owned warehouse (OW) has a fixed capacity of W units and the rented warehouse (RW) has unlimited capacity.
(9) The items of OW are consumed only after consuming the items kept in RW.
(10) The time of transporting items from RW to OW is ignored.

3. Development of the mathematical model

Let $Q(t)$ denote the system inventory level at time t, $(0 \leq t \leq T)$, the inventory level decreases according to demand as well as deterioration simultaneously. The change in inventory level can be represented using the following differential equation:

$$\frac{dQ(t)}{dt} + \lambda Q(t) = -D, \quad 0 \leq t \leq T \tag{1}$$

with the boundary condition $Q(T)=0$. The solution of Eq. (1) is

$$Q(t) = \frac{D}{\lambda}(e^{\lambda(T-t)}-1), \quad 0 \leq t \leq T \tag{2}$$

Additionally, the order quantity for each replenishment cycle is

$$Q = Q(0) = \frac{D}{\lambda}(e^{\lambda T}-1) \tag{3}$$

and the number of units that deteriorate per cycle is

$$Q-DT = \frac{D}{\lambda}(e^{\lambda T}-\lambda T-1) \tag{4}$$

This study observes that if order quantity $Q \leq \overline{W}$, the payment must be made immediately. Otherwise, the retailer will get a certain credit period, M. Clearly, the inequality $Q \leq \overline{W}$ holds if and only if $T \leq (1/\lambda)\ln((\lambda/D)\overline{W}+1) = T_{\overline{W}}$. Additionally, this study assumes that the retailer owns a warehouse with a fixed capacity of W units, meaning that any quantity exceeding this must be stored in a rented warehouse. From this perspective, this study observes that if the order quantity $Q \leq W$, renting a warehouse is unnecessary. Otherwise, W units of items are stored in the OW and the remainder are dispatched in the RW. Subsequently, the inequality $Q \leq W$ holds if and only if $T \leq (1/\lambda)\ln((\lambda/D)W+1) = T_a$. Herein, total annual variable cost function can be expressed as follows:

$$\begin{aligned} TVC(T) = &\ \textit{ordering cost} + \textit{purchasing cost} + \textit{deteriorating cost} \\ &+ \textit{stock-holding cost in RW} + \textit{stock-holding cost in OW} \\ &+ \textit{capital opportunity cost} + \textit{rental cost in RW}. \end{aligned}$$