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Evaluating lot-sizing strategies under limited-time price incentives: An efficient lower bound

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ABSTRACT

Determination of the optimal lot sizing strategy when the vendor offers limited time price incentives, such as pre-announcement of a price increase that will take effect after a finite time or a price discount that is valid for a limited time, is a common problem that has been extensively researched. A review of the literature indicates that the mathematical analysis and solution of this problem are quite complex. This complexity may deter managers from using the optimal strategy although an optimal lot sizing strategy results in the lowest cost. Managers generally prefer simple heuristic or rule-of-thumb strategies that are easy to understand and to implement, provided the total relevant cost associated with such strategies compares well with that of the optimal strategy. Therefore, it would be of significant value to managers if the cost associated with the optimal strategy can be deduced easily without recourse to complex mathematical analysis so that the simpler strategies can be quickly and easily evaluated. In this paper, we present an intuitively appealing and easy-to-compute method to determine a tight lower bound, whose value is very close to the total cost of the optimal strategy. We demonstrate, through extensive computational analysis, the adequacy of our lower bound by comparing it with the total cost associated with an optimal strategy over a wide range of operating parameters. Thus, managers can use it as a surrogate for the cost of the optimal strategy while evaluating heuristic strategies. We illustrate the application of our lower bound with numerical examples.

1. Introduction

It is commonly observed that vendors occasionally offer buyers some incentives that will remain effective only for a limited time. Such incentives may take several forms such as (i) a limited time reduction in the item's price, (ii) an interest-free delayed-payment privilege, or (iii) an advance notification of an imminent price increase. Vendors offer such incentives for various reasons such as a need to reduce excess inventories or to take up the slack in their production facility. Although a temporary price reduction, an advance notification of a price increase, and a temporary delayed-payment privilege are different types of limited-time price incentives, these are all conceptually the same and their analysis is similar. The importance and the wide range of interest in this problem are clearly evident from the extensive body of literature one finds on this topic. See Silver et al. (1998) for a detailed description of the problem.

1.1. Research context and motivation

This paper focuses on the problem of determining the optimal lot-sizing strategy for an inventoried item in the context of an infinite horizon economic order quantity (EOQ) model when the supplier offers a delayed-payment privilege for all orders placed from now on until a specified time in the future. This problem has been extensively researched (see Ramasesh (2010) for a detailed review of the literature). The following studies may be considered representative: Grubbström and Kingsman (2004) and Ramasesh and Rachamadugu (2001) present an analysis of the optimal policies for this problem using a discounted cash flow (DCF) approach. Lev and Weiss (1990) present an analysis of the optimal policies for this problem using an average cost approach.

The mathematical analysis of the problem and the solution procedures to find the optimal policies are extremely complex. Because of this complexity and the computational burden, practicing managers may find that the solution procedure for determining the optimal lot sizes difficult to understand and implement. Consequently, we feel that although an optimal lot sizing strategy results in the lowest cost, the complexity involved in its determination deters managers from using them. With the growing

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emphasis on supply chain management and coordination, the future problems are likely to become more complex. Managers prefer heuristic or rules-of-thumb strategies that are easy to understand and to implement, as long as the total relevant cost associated with such strategies compares well with that of the optimal policy as pointed out by Joglekar (2005). Therefore, it would be of significant value to managers if the cost associated with the optimal strategy can be ascertained easily without recourse to complex mathematical analysis. Development of intuitively appealing and easy-to-compute performance bounds would be of significant relevance for managerial practice. These considerations provide the motivations our research.

1.2. Objective and contribution to the literature

Our objective in this paper is to develop an intuitively appealing and easy-to-compute "lower bound", which is tight in the sense that the its value is very close to the total cost of the optimal solution to the infinite-horizon lot-sizing problem under limited-time price incentives. The significance and usefulness of such a tight lower bound comes from the following consideration: By comparing the total cost associated with heuristic or ad hoc policies with our lower bound on the optimal cost, we can evaluate simpler policies and make an informed choice without having to deal with complex (and often analytically intractable) mathematical analysis to seek optimal solutions and the associated computational burden.

The contribution of this paper to the literature is two-fold. First, it facilitates managerial policy implementation. Managers interested in adopting policies that include lot sizes based on heuristics or ad hoc considerations would find it useful to know how the total costs of these policies compare with the total costs from an optimal policy derived from a mathematically rigorous analysis. Since our lower bound is simple to compute and it is almost equal to the cost associated with the optimum policy, managers can use the lower bound as a benchmark for the evaluation of any heuristic or ad hoc lot-sizing policies. They may consider for evaluation a number of different heuristic policies and adopt a policy with costs acceptably close to the lower bound, which in turn is very close to the true optimum minimum cost. Second, it suggests an avenue for academic researchers to consider the development of intuitively simple and easy-to-compute heuristics and efficient bounds on the optimal solutions for a variety of complex problems in inventory management and related fields.

1.3. Organization of the paper

The rest of the paper is organized as follows. First, we describe the problem and explain why the mathematical analysis directed at seeking the optimal minimum cost solution is complicated. Then, we develop an expression for determining the value of the lower bound for any given lot-sizing situation without having to determine the optimal lot sizes through complex mathematical analysis. Next, we present computational results based on a detailed experimental design over a wide range of parameter values to establish the goodness of our lower bound by comparing it with the true optimal total cost based on the lot sizes derived from the application of the mathematically complex optimal lot-sizing procedure given by Ramasesh and Rachamadugu (2001). Finally, we illustrate the application of our lower bound to examine the performance of a popular heuristic strategy called the Early Purchase strategy and conclude with a summary of the implications of our contribution and its managerial significance.

2. Description of the problem and its complexity

Table 1 presents the notation used in this paper. For easy reference, we adopt the same notation as in Ramasesh and Rachamadugu (2001). The problem is to determine the optimal lot-sizing strategy for a stock-keeping unit (SKU) when the supplier offers an incentive by way of an interest-free credit period, τ, (this is equivalent to a reduction in the price/unit of the item from P to $Pe^{-r\tau}$) for all orders placed between now and a specified offer-expiration time in the future. The optimal strategy should minimize the present value of all relevant costs with the future costs discounted at a rate that represents the buyer's opportunity cost of capital.

Looking ahead from the time the vendor announces price incentive (call it time 0), the inventory manager's optimal lot-sizing strategy should specify three things: (1) Lot sizes in the interval $[0, T_p)$ during which the item price/unit is $Pe^{-r\tau}$, (2) Lot size at T_p, the last chance to order at the price/unit $Pe^{-r\tau}$, and (3) Lot sizes thereafter at price/unit P. For convenience and in line with the convention familiar to inventory managers, we measure inventory in terms of time units. Thus, if D is the demand per unit time, t "units of inventory" means Dt "units of the physical product."

If the item price is constant, then the classical EOQ formula [see Harris (1913)] can be used the optimal lot sizes for minimizing the relevant average cost per unit time over an infinite horizon. Also, simple EOQ-type formulas are available for determining the optimal order size when the inventory on hand is zero and the price discount is available only for the order placed now [see for example, Naddor (1966), Love (1979)].

But, in most real-life settings, the derivation of the optimal lot sizes over the infinite horizon is much more complex due to the following considerations:

(1) The parameters of the problem are non-stationary (i.e., price of the item is different) over the infinite time-horizon for cost minimization.
(2) The finite time horizon over which the price incentive is available may not generally be an integer multiple of the order cycles (except in rare situations due to pure chance).

Table 1
Notation.

D:	Demand per unit time, deterministic and constant.
P:	Regular price per unit of the item.
S:	Set up (or ordering) cost per set up.
h:	Holding cost per unit per period. This is exclusive of capital-related charges.
T_p:	Time at which delayed payment privilege or price discount expires.
τ:	Extended credit period or the time by which the payment for the orders placed during $[0, T_p]$ can be delayed without any interest payment.
r:	Discount rate for continuous compounding.
$\hat{T}$:	Optimum regular-reorder interval if the item price is constant at $Pe^{-r\tau}$ over the infinite horizon. This corresponds to the EOQ with item price $Pe^{-r\tau}$.
T^*:	Optimum regular-reorder interval if the item price is constant at P over the infinite horizon. This corresponds to the EOQ with item price P.
T_1^*:	Optimum size of the last, order (i.e., special order) placed at the lower price,$Pe^{-r\tau}$ in $[0, T_p]$.