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An integrated model for lot sizing with supplier selection and quantity discounts

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ABSTRACT

Good inventory management is essential for a firm to be cost competitive and to acquire decent profit in the market, and how to achieve an outstanding inventory management has been a popular topic in both the academic field and in real practice for decades. As the production environment getting increasingly complex, various kinds of mathematical models have been developed, such as linear programming, nonlinear programming, mixed integer programming, geometric programming, gradient-based nonlinear programming and dynamic programming, to name a few. However, when the problem becomes NP-hard, heuristics tools may be necessary to solve the problem. In this paper, a mixed integer programming (MIP) model is constructed first to solve the lot-sizing problem with multiple suppliers, multiple periods and quantity discounts. An efficient Genetic Algorithm (GA) is proposed next to tackle the problem when it becomes too complicated. The objectives are to minimize total costs, where the costs include ordering cost, holding cost, purchase cost and transportation cost, under the requirement that no inventory shortage is allowed in the system, and to determine an appropriate inventory level for each planning period. The results demonstrate that the proposed GA model is an effective and accurate tool for determining the replenishment for a manufacturer for multi-periods.

1. Introduction

Having a good production planning and replenishment control through effective inventory management is important for a firm to keep competitive in the market. The single product, multi-period inventory lot-sizing problem is one of the most common and basic problems, and it has often been tackled in the literature [1]. There are various extensions of the model to consider different issues in real environment. This research considers an environment with multiple periods and multiple suppliers. Different suppliers may have either all-units quantity discounts or incremental quantity discounts. In addition, transportation cost is fixed for each vehicle shipment, and it is different for different suppliers. By adopting the proposed models, the management can decide what quantity to order from which suppliers in which periods.

Quantity discount is a common and effective practice for suppliers to promote their products, and buyers can purchase products at a lower unit price when the ordering quantity is over a certain amount. In addition, multi-suppliers give firms a chance to purchase same materials from different sources. Ordering cost for each purchase can be in several different forms, for example, fixed, increasing or decreasing. The assumption of a fixed ordering cost is often seen in works [1–3]. In Rezaei and Davoodi [4], ordering cost consists of a fixed cost and an additional cost. The fixed cost is independent of the lot-size, and

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the additional ordering cost depends on the specific lot-size. In Rezaei and Davoodi [5], reduction in ordering cost is positively related to ordering frequency, that is, the higher the ordering frequency from a supplier, the higher the ordering cost reduction from that supplier. In this research, we set the ordering cost to be fixed because this is more suitable for our case study in real practice. In addition, quantity cost is considered in the study so that the more quantity ordered in each purchase, the lower the unit cost is.

Even though there have been abundant works which handled lot sizing problems in quantity discount environment and used MIP or other methods to solve the problems, the authors, after reviewing these papers, found out that very few papers have considered an environment with multiple periods, multiple suppliers, and both all-units and incremental quantity discounts simultaneously. With regard to the complexity of the lot-sizing problems, Rezaei and Davoodi [4,5], Florian et al. [6], and Bitran and Yanasse [7], claimed that this kind of problem basically belongs to a class of NP-hard problems. Therefore, to solve such a complicated problem, this paper proposes a MIP model to solve a small-scale problem and to compare the results with the proposed GA model first. The GA model is then used to tackle the problem when it becomes too complicated for the MIP model to solve. The proposed models are then compared with four past works that considered a similar problem environment. The comparisons show that the proposed models have more attributes than the past works.

The remaining of this paper is organized as follows. In Section 2, some related methodologies and works are reviewed. In Section 3, the problem under consideration and the assumptions are described. The formulation of the lot-sizing problem by MIP and the construction of the GA model are presented in Section 4. Case studies are carried out in Section 5. Section 6 compares the proposed models with several works with similar environments. Some conclusion remarks are made in the last section.

2. Related methodologies and works

2.1. Inventory management works

Intensive research has been carried out to develop models and methods and to find efficient solutions for better inventory management. Recent works of inventory management and lot sizing models are described below. Su and Wong [8] studied a stochastic dynamic lot-sizing problem under the bullwhip effect. A framework of two-stage ant colony optimization (TACO) was proposed, and a mutation operation was added in the second stage to determine the replenishment policy. Kim [9] analyzed the order quantity flexibility, under which a buyer provides to the supplier information about the expected future orders during a predetermined horizon and the supplier provides the buyer with the flexibility to adjust future orders in return. Kogan and Shnaiderman [10] studied a stochastic, optimal control problem characterized by continuous inventory replenishment, and proposed an asymptotically optimal, dynamic, base-stock policies that accounted for continuous inventory costs and continuous replenishment with periodic updates. Koo et al. [11] considered a capacity-constrained multiple-product system and developed a non-linear optimization model for simultaneous determination of throughput rate and lot size for each product. Since the optimization model could not be solved analytically, a line search algorithm for lot sizing at the capacity constrained workstation was proposed. Lu and Qi [12] studied a multi-product dynamic lot sizing problem, where inventories of all products are replenished jointly with the same quantity whenever a production occurs. A polynomial time algorithm was proposed to solve the problem without the option of lost sales, and two heuristic algorithms were presented to solve the problem with lost sales. Mohammadi and Jafari [13] presented a mathematical model for the problem of integrating lot sizing, loading, and scheduling in a capacitated flexible flow shop with sequence-dependent setups, and three mixed integer programming-based algorithms were proposed to solve the model. Okhrin and Richter [14] analyzed an uncapacitated single-item lot sizing problem where a minimum order quantity restriction guarantees a certain level of production lots. A solution algorithm based on the concept of atomic sub-problems was proposed, and the $O(T^2)$ exact algorithm with the worst-case complexity could solve the problem to optimality. Okhrin and Richter [15] further explored a single-item capacitated lot sizing problem with minimum order quantity, and adopted a general dynamic programming technique to develop an $O(T^3)$ exact algorithm to optimally solve the problem. Tarim et al. [16] proposed a computational approach to solve the mixed integer programming (MIP) model of a stochastic lot-sizing problem with service level constraints under static-dynamic uncertainty strategy. The approach was based on a relaxation of the MIP model, and an optimal solution to the relaxed model was found by solving an equivalent shortest path problem. Ho et al. [17] investigated the multi-stage logistics and inventory problem in an assembly type supply chain in which a uniform lot size is produced through all stages and unequal sub-batch sizes across stages are allowed. A serial-type supply chain model is constructed first, an assembly-type supply chain model is established next, and finally an optimization algorithm that incorporates the lot size division and recursive tightening methods is developed to solve the problems.

2.2. Quantity discounts

Multiple suppliers and quantity discounts are common practices in purchasing activities. When quantity discounts is present and a large order is placed by a buyer, the unit price charged by the supplier is reduced according to a predetermined schedule. There are often a number of price breaks, and the unit discounted price decreases as the quantity level increases. Two major types of quantity discounts are present: all-units quantity discount and incremental quantity discount [18,19].