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Joint optimal lot sizing and production control policy in an unreliable and imperfect manufacturing system

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ABSTRACT

This paper deals with the problem of the joint determination of the optimal lot sizing and optimal production control policy for an unreliable and imperfect manufacturing system, where the quality control of lots produced is performed using an acceptance sampling plan. The proportion of defective items, the time between failures and the time to repair are generally distributed. The incurred total cost includes manufacturing cost, transportation cost, inspection costs, rejection cost of defective items, replacement cost for returned defective items from customers, and holding and backlog costs. The associated cost minimization problem is formulated with a stochastic dynamic programming model where the lot sizing and production rate are considered as decision variables. Given the difficulties in solving such a highly stochastic model analytically or numerically, we adopted a modified hedging point policy (HPP) to control the production rate, as well as an economic lot sizing policy for batch processing control; we also relied on a simulation-based experimental approach to determine a close approximation of the optimal control parameters. It is shown that production should be accelerated at the maximum production rate, not only when building the safety stock, as in the classical HPP, but also after rejecting a lot, in order to recuperate the loss in inventory and to maintain the on-hand safety stock. Numerical experiments and thorough sensitivity analyses are provided to illustrate the effectiveness of the proposed control policy and the robustness of the resolution approach. Some interesting behaviours regarding the impact of different parameters on the optimal decision variables are observed and discussed.

1. Introduction

Manufacturing systems are subject to many stochastic phenomena such as random failures and repairs, imperfect production, quality deterioration, etc. Feedback control policies have proven their effectiveness in responding to such random events (Gershwin, 1994). For continuous-time failure-prone production systems, it has been shown that the well-known hedging point policy (HPP) is optimal (Akella and Kumar, 1986; Bielecki and Kumar, 1988). This feedback policy controls the production rate as a function of instantaneous operational system state and inventory level, which consists in building and maintaining a safety stock during periods of system availability in order to hedge against future capacity shortages brought about by failures.

For batch manufacturing systems, the economic production quantity (EPQ) model is often used for production-inventory control, and to determine the optimal lot sizing that minimizes overall incurred costs. Research on the EPQ model has been undertaken in different contexts of reliability and/or quality imperfection, especially over the last two decades. In one of the pioneering papers that addressed the EPQ problem of unreliable batch manufacturing systems, Groenevelt et al. (1992a) investigated the impact of system breakdowns and corrective maintenance on production lot sizing decisions. Assuming a deterministic constant production rate, negligible repair time, exponential failures and no backlogs, the authors determined the optimal lot sizing for two production reorder policies (no-resumption (NR) policy and abort/resume (AR) policy). In a subsequent study, Groenevelt et al. (1992b) defined a production control policy to simultaneously determine the optimal lot sizing and the safety stock level that satisfy a prescribed service level. They assumed that during a production run, a certain fraction of the items produced is instantaneously diverted into the safety stock. Kim and Hong (1997) and Kim et al. (1997) extended the Groenevelt et al. (1992a) model, which assumes that the times between failures follow general distributions. Chung (1997) determined an approximate formula for the optimal lot sizing of the Groenevelt et al. (1992a) model by calculating its bounds. Giri et al. (2005) focused on the problem of EPQ for an unreliable production system where the production rate is treated as a decision variable. They developed two models: with and without safety stock, in order to jointly determine the optimal lot sizing and the optimal feasible

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production rate. Giri and Dohi (2005) extended the Giri et al. (2005) model with safety stock, taking into account preventive maintenance, and assuming that the failure and repair times are general distributions. Bouslah et al. (2013) obtained an integrated optimal lot sizing and feedback production policy, considering a transportation delay of lots produced and added to the serviceable stock. All the above-cited studies deal with the effect of process reliability on the EPQ model, and do not consider the quality issue, instead, assuming that all produced items are of perfect quality.

On the other hand, many works have considered the quality imperfection problem in the EPQ model, without reliability. Porteus (1986) and Rosenblatt and Lee (1986) were among the first researchers who investigated the effect of quality imperfection on the EPQ. In both studies, they assumed that the deterioration of production system is a random process characterized by two states: the 'in-control' state, when all items produced are of conforming quality, and the 'out-of-control' state, when some percentage of items produced are defective. Lee and Rosenblatt (1987) considered the maintenance by inspection feature to monitor deterioration of the production process: if inspections indicate that the production process is 'out of control', it will be restored to the 'in-control' state. They then focused on simultaneously determining the EPQ and optimal inspection schedules. Khouja and Mehrez (1994) formulated an EPQ model assuming that production rate is a decision variable and that the quality of the production process deteriorates with an increased production rate. Salameh and Jaber (2000) presented a modified inventory model which extends the traditional EPQ model by accounting for imperfect quality items. Hayek and Salameh (2001) derived an optimal operating policy for an EPQ model under the effect of imperfect quality. They assumed that all defective items produced are reworked and added to perfect quality inventory, and that shortages are allowed and backordered. Ben-Daya (2002) developed an integrated model for the joint determination of the EPQ and preventive maintenance level for an imperfect process having a general deterioration distribution with an increasing hazard rate. Chiu (2003) extended the Hayek and Salameh (2001) model, by assuming that not all of the defective items produced are reworked, and that a portion of all imperfect quality items are scrapped and discarded before the rework process is started. Finally, Sana (2010) considered that the percentage of defective items varies linearly with both the production rate and the production-run time, and that the probability distribution of the shift time from the 'in-control' to the 'out-of-control' state also depends on the production rate. Therefore, he focused on determining the optimal lot sizing and the optimal production rate.

In the aforementioned EPQ models, reliability and quality issues are studied separately. However, these two problems are often observed simultaneously in real-life manufacturing systems. Only few recent EPQ models jointly consider the effects of equipment breakdowns and quality deterioration in the production process. Among these works, Chiu et al. (2007) extended the works of Chung (1997) and Chiu (2003) in order to determine the optimal run time problem of EPQ models with scrap, reworking of defective items, and stochastic machine breakdowns. Liao et al. (2009) integrated maintenance programs (perfect/imperfect preventive maintenance and imperfect repair) with the EPQ model for an imperfect and unreliable manufacturing system. Chakraborty et al. (2009) developed integrated production, inventory and maintenance models in order to study the joint effects of process deterioration, machine breakdown and inspections on optimal lot sizing decisions. Sana and Chaudhuri (2010) extended the Giri and Dohi (2005) model, considering the effect of an imperfect production process subject to random breakdowns and variable safety stocks. The proposed policy consists in determining the optimal safety stock, the optimal production rate and the optimal lot sizing.

In most existing EPQ models, the effects of using such a quality control policy on the production policy parameters (including lot sizing) have not been sufficiently studied. Indeed, inspection is considered only as a tool for controlling the quality deterioration of the production process. As well, most models assume (except for a few works like Ben-Daya (2002)), that the inspection delay is negligible. However, inspection is in itself an important part of quality assurance, which should be fairly represented in the EPQ model. Some authors, such as Salameh and Jaber (2000), assume that all lots produced are 100% inspected. Liao et al. (2009) consider a complete quality audit using automated inspection. From an economic point of view, the cost of a 100% inspection is very high, particularly with automation systems requiring high technology. According to Chin and Harlow (1982), the inspection process is normally the largest single cost in manufacturing.

In manufacturing organizations, statistical techniques, such as control charts and acceptance sampling plans, can be used for quality control when the cost of a 100% inspection is higher than the cost of delivering a certain proportion of defective items (Besterfield, 2009). Only few researchers have integrated quality control techniques into EPQ models such as Ben-Daya (1999) who presented an integrated model for the joint optimization of production quantity, design of quality control parameters using the $\bar{x}$-control chart and the maintenance level.

Another common assumption made in most EPQ models is that the lot which is currently being processed can instantly meet the demand, and even build a safety stock if the production rate is strictly greater than the demand rate. This assumption is unrealistic for a wide range of manufacturing systems where a certain delay, for lot sampling, inspection, reworking, etc., exists between the production and the final stock that truly serves the demand.

Considering the limitations of past models in the context of quality and production-inventory control, the main purpose of this paper is to develop an integrated optimal lot sizing model and a feedback control policy for unreliable and imperfect batch manufacturing systems, where quality control is performed using a single acceptance sampling plan. The problem is formulated using a stochastic dynamic programming in which the lot sizing and the production rate are considered as decision variables. Since it is difficult to analytically or numerically derive the optimal solution of the model, we adopt a modified HPP to control the production rate and a feedback-inventory to control the batch processing order using theoretical arguments and approximations. Then, we use a simulation-based experimental approach to determine the optimal parameters of the proposed control policy, when the proportion of defective items, and failure and repair times follow general distributions.

The remainder of this study is organized as follows. Section 2 introduces the notation and the problem description, and presents a formulation of the optimal production problem. Section 3 presents the proposed control policy. Section 4 describes the resolution approach used to determine the optimal design of the control policy and the optimal incurred cost. Section 5 provides an illustrative numerical example of the resolution approach. A thorough sensitivity analysis is given in Section 6, in order to confirm the robustness of the resolution approach and to study the behaviour of the system under different settings of cost and quality parameters. Finally, Section 7 concludes this paper.

2. Problem formulation

2.1. Notation

The following notation will be used throughout the paper:

$q(t)$ WIP lot level at time t (units)
$x(t)$ inventory level at time t